NONLINEAR SKIN EFFECT AND ELECTROMAGNETIC SOUND GENERATION

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The electromagnetic generation of longitudinal sound in metals under conditions of the nonlinear anomalous skin effect, when the main source of acoustic oscillations is the deformation force, is investigated theoretically. The nonlinearity mechanism is magnetodynamic: it is related with the influence of the radio-wave magnetic field on conduction-electron dynamics. The analysis is carried out in a broad range of values of the external-signal amplitude $\mathscr{H}$, and covers both weak and strong nonlinearities. The dependence of the amplitude of generated sound on the radio-wave amplitude $\mathscr{H}$, its frequency ω, and mean free path l, is derived. It is established that the longitudinal sound contains only even harmonics of the incident wave. Nonlinear transformation of electromagnetic energy into acoustic energy is little sensitive to the state of the sample surface.

1. A large number of works has been dedicated to the theoretical and experimental investigation of the contactless sound generation (see, for example, the review [1] and references therein). The majority of these researches cover only the linear regime where the external electromagnetic wave of frequency ω has a small amplitude $\mathscr{H}$, and, therefore, generates acoustic oscillations of the same frequency in the metal. The efficiency of electromagnetic sound generation grows as the incident-signal power and conduction-electron mean free path l are increased. However, with increasing the parameters $\mathscr{H}$ and l, the nonlinear processes in the sample begin rapidly enough to develop. In the modern experiments with clean metals at low temperatures, precisely, the nonlinear regime turns out to be the commonest.

The nonlinear electromagnetic generation of acoustic oscillations under conditions of normal skin effect, $l \ll \delta$ (δ is the skin depth), was theoretically studied in refs. [2,3]. The situation of anomalous skin effect

$$\delta \ll l \tag{1}$$

which is typical for metals, is of great interest. This case was investigated only in one theoretical paper [4] that covers the weak-nonlinearity regime. The results of ref. [4] demonstrate the importance of nonlinear electromagnetic processes in the sound-generation effect. Thus, for instance, in metals with spherical Fermi surface the longitudinal-sound generation is due only to nonlinearity. In this case the amplitude of nonlinear ultrasound is $(l/\delta)^2 \gg 1$ larger than under normal skin effect.

In metals the nonlinearity mechanism is related with the influence of the radio-wave magnetic field on the electron dynamics, and, therefore, on the sample conductivity. This mechanism is called magnetodynamic. The parameter b, which characterizes the effectiveness of the mechanism, is defined by the ratio of the mean free path l to the electron-trajectory length in the skin-layer heterogeneous magnetic field $(8R\delta)^{1/2}$ (see ref. [5]):

$$b = (2\mathscr{H}/h)^{1/2}, \quad h = 8cp_F\delta/el^2 . \tag{2}$$

Here $R = cp_F/2e\mathscr{H}$ is the characteristic curvature radius of the electron trajectory in the skin-layer magnetic field, e the absolute value of the charge, p_F the Fermi momentum, and c the velocity of light. Let us estimate the quantity h with the nonlinearity parameter $b=1$. For typical pure metals at low temperatures, $\delta \sim 10^{-4}$–10^{-3} cm and $l \sim 10^{-1}$ cm, we get $h \sim 0.5$–5 Oe. In experiments the value of the electromagnetic-wave amplitude $\mathscr{H}$ reaches a few tens, and, inclusive, hundred of oersteds, and then both the case of weak nonlinearity ($b \ll 1$) and the strong-nonlinearity case ($b \gg 1$) are experimentally realizable.

2. The kinetic theory of nonlinear anomalous skin effect was formulated in ref. [5]. We will comment the results of this work briefly.

In the case of weak nonlinearity ($b \ll 1$) the electron trajectories in the skin layer are almost straight lines, being slightly curved by the wave magnetic field. In these conditions the nonlinear effects manifest themselves in a quadratic approximation in the amplitude $\mathscr{H}$ ($b^4 \propto \mathscr{H}^2$). Therefore, in the leading approximation in b^4 the skin effect is described by the linear theory, and the surface impedance Z does not depend on the amplitude $\mathscr{H}$. The dependence of Z on $\mathscr{H}$ appears only in the next terms of impedance expansion in powers of the parameter b^4:

$$[Z(\mathscr{H})-Z(0)]/Z(0) \sim b^4 . \tag{3}$$

In the strong-nonlinearity regime ($b \gg 1$) the electrodynamic properties of the metal are attributed to a group of trapped electrons [5]. This group appears due to the spatial distribution of the radio-wave magnetic field being sign-variable. The trapped electrons move along the sample surface in trajectories which wind around the plane where the magnetic field changes its sign (see fig. 1). They remain the whole time in the skin layer, and, therefore, interact most effectively with the electromagnetic wave. Consequently, in the conditions of strong nonlinearity the skin effect is determined by trapped electrons. The relative number of such electrons is of the order of $(\delta/R)^{1/2}$, and, according to the inequality $b \gg 1$, it exceeds the relative number of effective electrons in the linear theory, δ/l. For this reason, in the developed nonlinearity ($b \gg 1$) the metal conductivity increases considerably, and then the skin depth δ and surface impedance decrease.

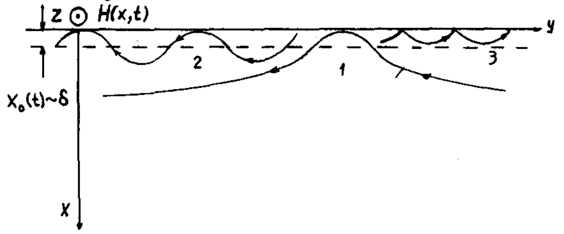

Fig. 1. Trajectories of effective electrons in the magnetic field of the electromagnetic wave: transient (1), trapped (2), and skipping electrons (3). $x = x_0(t)$ is the plane where the magnetic field $H(x, t)$ changes its sign, $H(x_0, t) = 0$.

The dependence of δ and Z on the amplitude $\mathscr{H}$ is easily derived with the aid of Pippard's ineffectiveness concept. Within this model the conductivity of trapped electrons has the form

$$\sigma = \sigma_0 \left(\frac{\delta}{R}\right)^{1/2} = \sigma_0 \frac{\delta}{l} b , \tag{4}$$

where σ_0 is the massive-metal static conductivity. From the Maxwell equations the relation between the skin depth δ and effective conductivity σ is expressed as

$$\delta^2 = \mathrm{i}c^2/4\pi\omega\sigma . \tag{5}$$

Substituting (2), (4) into (5), and solving the resulting equation for δ, we obtain

$$\delta = \left[\left(\frac{c^5 p_F}{4\pi^2 e\omega^2\sigma_0^2 \mathscr{H}}\right)\right]^{1/5} e^{\mathrm{i}\pi/5} ,$$

$$Z = -\frac{4\pi\mathrm{i}\omega\delta}{c^2} \propto \mathscr{H}^{-1/5} . \tag{6}$$

Thus, in the strong-nonlinearity regime ($b \gg 1$) the surface impedance of the metal depends on the incident-wave amplitude already in the leading approximation. Furthermore, it follows from (6) and (3) that the effect of the quantity $\mathscr{H}$ on the surface impedance becomes notable even earlier: in the transition region where the parameter b is of the order of unity and the external-signal intensity is relatively low.

Note that the dependence of the impedance Z on the amplitude $\mathscr{H}$ and other parameters of the problem is not sensitive to the character of the reflection of electrons from the metal surface. Indeed, trapped particles, which determine the nonlinear skin effect, do not interact with the sample boundary absolutely. When the reflection is specular, in addition to trapped particles, a group of skipping electrons appears (see fig. 1). Nevertheless, their relative number and contribution to the conductivity are of the same order of magnitude as the contribution of the trapped particles.

It is an interesting characteristic of the nonlinear anomalous skin effect that the electromagnetic field in the metal does not contain even harmonics of the incident wave. The generation of harmonics occurs because of the dependence of conductivity on time. The conductivity, which is a functional of the modulus of the magnetic-field strength, has a period π/ω, and contains only even harmonics. Hence, the incident wave of frequency ω generates only odd electromagnetic-field harmonics in the metal.

3. Let us consider a metallic semispace on whose surface a plane, monochromatic, electromagnetic wave of frequency ω and amplitude $\mathcal{H}$ is incident. We will orient the x axis along the normal into the interior of the metal ($x=0$ at the boundary) and the y and z axes parallel to the electric and magnetic components of the electromagnetic field (see fig. 1):

$$E(x,t)=\{0,E(x,t),0\}\,,$$
$$H(x,t)=\{0,0,H(x,t)\}\,, \tag{7}$$

We investigate the electromagnetic generation of a longitudinal acoustic wave in which the vector of displacements is

$$\boldsymbol{u}(x,t)=\{u(x,t),0,0\}\,. \tag{8}$$

The system of equations, which describes this process, is composed of the Maxwell equations, Boltzmann's kinetic equation for the electron distribution function, and the elasticity-theory equation for the displacement field $\boldsymbol{u}(x,t)$. In this system of equations we will despise the small terms that are proportional to some power of $m/M\ll 1$ (m is the electron mass, M is the ion mass). This implies that the solution of the above mentioned system is divided into two stages. In the first stage one resolves the problem about the nonlinear perturbation of the electronic subsystem by an external electromagnetic wave, and about the distribution of the fields $E(x,t)$ and $H(x,t)$ in the metal. This stage, in fact, is the problem about the nonlinear anomalous skin effect, resolved in ref. [5]. The nonlinearity is related with the magnetic field $H(x,t)$, and contained in the Lorentz force of the kinetic equation. We will limit our attention to the quasistatic case

$$\omega\ll\nu\,, \tag{9}$$

when the external-wave frequency ω is considerably smaller than the electronic relaxation frequency ν. Inequality (9) permits us to neglect the variation of the magnetic field $H(x,t)$ during the whole mean free time of the electrons.

The effect of longitudinal-sound generation is owing to a force $F(x,t)$ in the equation of longitudinal vibrations of the lattice. This force is produced by the electrons:

$$F(x,t)=\frac{\partial}{\partial x}\left(\int\frac{2\,d\boldsymbol{p}}{(2\pi\hbar)^3}\Lambda_{xx}(\boldsymbol{p})\,\frac{\partial f_F}{\partial\epsilon}\chi\right), \tag{10}$$

Here $(\partial f_F/\partial\epsilon)\chi$ is the nonadiabatic addition to the Fermi-distribution function f_F and $\boldsymbol{p}$ is the electron momentum. In the model of electronic dispersion law that we will use the component $\Lambda_{xx}(\boldsymbol{p})$ of the deformation-potential tensor has the form

$$\Lambda_{xx}(\boldsymbol{p})=-\tilde{m}(v_x^2-v_F^2/3)\,, \tag{11}$$

where $\tilde{m}$ is the "deformation" mass, $\boldsymbol{v}=\boldsymbol{p}/m$ the electron velocity, and v_F the Fermi velocity. In the expression for the force (10) we considered only the deformation mechanism of the electron–phonon interaction. The analysis shows that in conditions of anomalous skin effect the induction contribution to $F(x,t)$ is negligible independent of the degree of nonlinearity. In the weak-nonlinearity case ($b\ll 1$) the induction contribution to $F(x,t)$ *is* $(\delta/l)^2\ll 1$ times smaller than the deformation contribution, and in the strong-nonlinearity case ($b\gg 1$) it is $\delta/R\ll 1$ time smaller.

It is not difficult to solve the equation of acoustic vibrations after representing the displacement $u(x,t)$ and force $F(x,t)$ in the form of a Fourier series:

$$u(x,t)=\frac{1}{\pi}\sum_{n=-\infty}^{\infty}e^{-in\omega t}\int_0^\infty dk\sin(kx)\,\tilde{u}_{n\omega}(k)\,,$$
$$F(x,t)=\frac{1}{\pi}\sum_{n=-\infty}^{\infty}e^{-in\omega t}\int_0^\infty dk\sin(kx)\,\tilde{F}_{n\omega}(k)\,. \tag{12}$$

In the region $x\gg l$ the solution assumes the form of plane-wave superposition:

$$u(x,t)=\sum_{n=-\infty}^{\infty}u_n\exp[in(qx-\omega t)]\,,$$
$$q=\omega/s,\quad x\gg l\,, \tag{13}$$

where s is the velocity of the longitudinal sound, the amplitude of the nth harmonic is determined by the expression $(n\neq 0)$

$$u_n = \frac{\mathrm{i}}{n\pi\rho_0\omega s}\int_0^\infty \mathrm{d}k \frac{k}{k^2-(nq)^2}\tilde{F}_{n\omega}(k)\,, \qquad (14)$$

ρ is the metal density. In the course of the derivation of (13) and (14) we have considered that the attenuation-sound length l_s is the largest parameter in the problem. Indeed, when the electron–phonon interaction is not resonant, $l_s \gg \delta$, $l_s \sim v_F/sq \gg l/q$, and $l_s \sim lv/\omega \gg l$.

In conclusion, the problem about electromagnetic sound generation reduces to the computation of the amplitudes u_n (eq. (14)).

4. In the present work with the aid of a detailed analysis of conduction-electron dynamics in a radiowave heterogeneous magnetic field the kinetic equation for χ was solved, and a general expression for the deformation force was obtained. On the basis of this expression we could examine both weak and strong nonlinearities from the same point of view. Moreover, the dependence of the generated-sound amplitude on the external-signal amplitude $\mathscr{H}$, its frequency ω, mean free path l, and other parameters of the problem, was derived. It is shown that the generated longitudinal sound contains only even harmonics (2ω, 4ω, ...) of the incident wave. It is established that the nonlinear electromagnetic generation of acoustic vibrations is little sensitive to the state of the sample surface.

For lack of space we will exhibit only the interpolation function which describes the dependence of the second-harmonic amplitude $(n=2)$ of the produced longitudinal sound on the quantity $\mathscr{H}$:

$$|u_2| = \frac{2}{3\pi^4}\frac{\tilde{m}}{m\rho_0\omega s}\left(\frac{cp_F}{el}\right)^2 b^4[1-\mathrm{e}^{-1/b}]^2 G(q\delta)\,, \qquad (15)$$

$$G(x) = \frac{1+x^2[(x-1)\ln x+2]}{1+x^3(x-1)\ln x} \times\left[1+\left(\frac{1+x^3}{1+x^{5/2}}\right)^{(b-1)/(b+1)}\right]. \qquad (16)$$

This formula was constructed on the basis of the found, asymptotically exact, expressions for the sound amplitude in the cases of weak $(b\ll 1)$ and strong $(b\gg 1)$ nonlinearities. It is valid for arbitrary values of the parameters b and $q\delta$, being accurate up to a constant. The depth of the electromagnetic-field penetration, δ in eq. (15), is determined by resolving the equation

$$\delta = \delta_a[1-\exp(-1/b)]^{1/3}\,, \qquad (17)$$

where δ_a is the skin thickness of the linear theory:

$$\delta_a = (c^2 l/3\pi^2\sigma_0\omega)^{1/3}\,. \qquad (18)$$

Let us analyze the dependence of the amplitude u_2 on the quantity $\mathscr{H}$ in the cases of small and large values of the nonlinearity parameter b (2). In the weak-nonlinearity case $(b\ll 1)$ the terms with $\mathrm{e}^{-1/b}$ can be omitted from expressions (15) and (17). Subsequently, the depth δ coincides with δ_a, and the amplitude $|u_2|$ is proportional to b^4 $(b^4 \propto \mathscr{H}^2)$, that is, to the square of the quantity $\mathscr{H}$.

Note that in conditions of weak nonlinearity the expansion parameter is $b^4 \ll 1$. For this reason the even harmonics with $|n|>2$ are small compared to the second harmonic $(n=2)$ as $b^{2|n|-4} \ll 1$.

In conditions of strong nonlinearity $(b\gg 1)$ the dependence of $|u_2|$ (15) on the amplitude $\mathscr{H}$ is determined by the parameter $q\delta$, since with $b\gg 1$ the depth δ is proportional to $\mathscr{H}^{-1/5}$ $(\delta \propto \mathscr{H}^{-1/5})$. In the case $q\delta \ll 1$ the function $G(q\delta)$ is equal to $G(0)=2$ in the leading approximation. Then for the amplitude u_2 (15) we find

$$|u_2| \sim \frac{\mathscr{H}^2}{\rho_0\omega s}\frac{R}{\delta} \propto \mathscr{H}^{6/5}\omega^{-3/5}l^{2/5}\,. \qquad (19)$$

In the opposite situation, when the sound wavelength is considerably smaller than the skin thickness, $q\delta \gg 1$, the function $G(q\delta)$ is of the order of $1/(q\delta)^{1/2}$. Hence

$$|u_2| \sim \frac{\mathscr{H}^2 R}{\rho_0\omega s\delta}\frac{1}{(q\delta)^{1/2}} \propto \mathscr{H}^{13/10}\omega^{-9/10}l^{3/5}. \qquad (20)$$

The case of strong nonlinearity is characterized by the fact that the amplitudes of all even harmonics are quantities of the same order. In this case the group of trapped electrons gives the fundamental contribution to those amplitudes.

5. At low temperatures and in conditions of quasistatic ($\omega \ll \nu$) anomalous skin effect the case $q\delta \ll 1$ ($\omega \ll s/\delta$) is of great interest. Formula (19), which is valid in this case, is easily obtained by making a simple analysis. Indeed according to (14), when $q\delta \ll 1$ the amplitude of the generated second harmonic is described by the following qualitative formula ($k \sim \delta^{-1}$):

$$|u_2| = F\delta/\rho_0 \omega s \,. \tag{21}$$

From (10) and (11) we can derive the relation between the deformation force F and the current density j:

$$F \sim jmv_F/e\delta \,. \tag{22}$$

The expression for the current density j is

$$j = \sigma E, \quad E \sim \mathscr{H}\omega\delta/c \,, \tag{23}$$

and the conductivity σ and skin depth δ can be determined using Pippard's ineffectiveness concept. After using expressions (4), (6), (21)–(23), we get the quantity $|u_2|$ in the form (19).

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