HYSTERESIS AND JUMPS FOR THE AMPLITUDE OF ELECTROMAGNETICALLY EXCITED SOUND IN METALS PLACED IN A MAGNETIC FIELD

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Received 15 September 1988; accepted for publication 27 September 1988
Communicated by V.M. Agranovich

The electromagnetic generation of longitudinal sound in current states of metals is investigated theoretically. The dependence of the amplitude of the generated sound on the radio-wave amplitude $\mathscr{H}$, its frequency ω, external constant magnetic field h_0, electronic mean free path l and other problem parameters, is derived. It is shown that the generated longitudinal sound contains the full set of multiple harmonics (ω, 2ω, 3ω, ...) of the incident wave independently of the degree of nonlinearity. It is extablished that the harmonics amplitudes of the excited sound, as functions of the external constant magnetic field h_0, have a hysteresis behavior. The shape variation of the hysteresis loops for the acoustic harmonics amplitudes, with increasing amplitude $\mathscr{H}$, is studied. Under conditions of developed nonlinearity, sharp changes of the longitudinal sound amplitude in time were found.

1. In the last few years a large number of experimental and theoretical works, devoted to the investigation of nonlinear electromagnetic phenomena in metals, has been published (see, for example refs. [1–15]). In clean metals at low temperatures the magnetodynamic nonlinearity mechanism, which is related with the influence of the magnetic wave component on electron trajectories' form and, consequently, on the high-frequency conductivity of the sample, is the most effective.

In the commonest situation (anomalous skin effect) for metals, when the skin depth δ is much smaller than the electron mean free path l and the radius of the curvature R for the electron trajectories in the wave magnetic field,

$$\delta \ll l, \quad R = cp_F/2e\mathscr{H}, \qquad (1)$$

the magnetodynamic nonlinearity is characterized by the parameter b:

$$b = (2\mathscr{H}/\mathscr{k})^{1/2}, \quad \mathscr{k} = 8cp_F\delta/el^2. \qquad (2)$$

Here $\mathscr{H}$ is the radio-wave amplitude, p_F the Fermi momentum, e the absolute value of the charge, c the velocity of light. The quantity b represents the ratio of the mean free path l to the electron trajectory length in the skin layer $(8R\delta)^{1/2}$. For typical metals the fields' $\mathscr{k}$, in which the electron trajectory bends become appreciable and the magnetodynamic nonlinearity mechanism begins to manifest itself, has a small magnitude of 0.5–5 oersted.

The magnetodynamic nonlinearity plays an important role in the process of electromagnetic sound generation. Thus, for instance, in the absence of the external constant magnetic field the generation of longitudinal sound in isotropic metals is due exclusively to nonlinearity [16,17]. In this case the generated sound contains only even harmonics of the incident wave. In ref. [17] it was shown that the dependence of the sound amplitude on the radio-wave amplitude $\mathscr{H}$, the frequency ω, and the electron mean free path is absolutely determined by characteristics of the nonlinear anomalous skin effect [2].

It is necessary to note that in the presence of the external constant magnetic field h_0 the picture of the nonlinear effects in metals becomes quite rich and very interesting. According to this, the inclusion of the field h_0 should lead to new curious properties for the amplitude of the generated sound. One of the most important and interesting nonlinear effects, observed in the field $|h_0| \leqslant 2\mathscr{H}$, is the excitation of current states [13,14]. When a metal is irradiated by radio waves of sufficiently large amplitude, a rectified current and a constant heterogeneous magnetic field appear in the sample. It is noteworthy that the value h of the induced field for the sample depth, as a function of the external constant magnetic field h_0, has a hysteresis behavior. The theory that describes the hysteresis $h(h_0)$ was formulated in refs. [9–13].

The present work is devoted to the theoretical analysis of the nonlinear electromagnetic sound excitation in metals in the current state. The dependence of the amplitude of the generated sound on the amplitude of the external signal $\mathscr{H}$, its frequency ω, the field h_0, the mean free path l and other parameters of the problem is first derived here. It is shown that the generated longitudinal sound contains the full set of multiple harmonics (ω, 2ω, 3ω, ...) of the incident wave indepently of the degree of nonlinearity. It is established that the harmonics amplitudes of the excited sound, as functions of the external constant magnetic field h_0, have a hysteresis behavior. The shape variation of the hysteresis loops for the acoustic harmonics amplitudes, with increasing amplitudes $\mathscr{H}$, is studied. In conditions of developed nonlinearity ($b \gtrsim 1$) we found sharp changes of the amplitude of the longitudinal sound in time, which are related with jumps of the radio-wave electric field on the sample boundary [11,12].

2. Let us consider a plane monochromatic electromagnetic wave with frequency ω and amplitude $\mathscr{H}$ incident on the surface of a metallic semispace. We will orient the x axis along the normal to the surface into the interior of the sample, and the y and z axes parallel to the vectors of the electric and magnetic wave fields:

$$\boldsymbol{E}(x,t)=\{0,E(x,t),0\}, \quad \boldsymbol{H}(x,t)=\{0,0,H(x,t)\}. \tag{3}$$

The external constant magnetic field $\boldsymbol{h}_0$ is oriented collinearly to the vector $\boldsymbol{H}(x,t)$.

We investigate the electromagnetic generation of a longitudinal acoustic wave in which the vector of displacement is

$$\boldsymbol{u}(x,t)=\{u(x,t),0,0\}. \tag{4}$$

The system of equations which describes this process, is composed of Maxwell's equations, Boltzmann's kinetic equation for the electron distribution function, and the equation of elasticity theory for the displacement field $\boldsymbol{u}(x,t)$. In this system of equations we neglect the small terms that are proportional to some power of $m/M \ll 1$ (m is the electron mass, M is the ion mass). The Boltzmann equation is linearized in the electric field $E(x,t)$. The nonlinearity is related with the magnetic field $H(x,t)$, and contained in the Lorentz force of the kinetic equation. We will limit our attention to the quasistatic case

$$\omega \ll \nu, \tag{5}$$

where ν denotes the electronic relaxation frequency. Inequality (5) permits us to neglect the variation of the magnetic wave field $H(x,t)$ during all mean free times of the electrons.

It is not difficult to solve the equation of acoustic vibrations after representing the displacement $u(x,t)$ and the excitatory force $F(x,t)$ in the form of Fourier series:

$$u(x,t)=\frac{1}{\pi}\sum_{n=-\infty}^{\infty} \mathrm{e}^{-\mathrm{i}n\omega t}\int_0^\infty \mathrm{d}k\,\sin(kx)\tilde{u}_{n\omega}(k),$$

$$F(x,t)=\frac{1}{\pi}\sum_{n=-\infty}^{\infty} \mathrm{e}^{-\mathrm{i}n\omega t}\int_0^\infty \mathrm{d}k\,\sin(kx)\tilde{F}_{n\omega}(k). \tag{6}$$

In the region $x \gg l$ the solution assumes the form of a superposition of plane waves:

$$u(x,t)=\sum_{n=-\infty}^{\infty} u_n \exp[\mathrm{i}n(qx-\omega t)],$$

$$q=\omega/s, \quad x \gg l, \tag{7}$$

where q is the wave number, s is the velocity of the longitudinal sound, and the amplitude of the nth harmonic is determined by the expression:

$$u_n=\frac{\mathrm{i}}{n\pi\rho_0\omega s}(nq)^2 \fint_0^\infty \mathrm{d}k\,\frac{1}{k^2-(nq)^2}\frac{\tilde{F}_{n\omega}(k)}{k},$$

$$n \neq 0. \tag{8}$$

Here ρ_0 is the metal density, $\tilde{F}_{n\omega}(k)$ denotes the Fourier component of the deformation force $F(x,t)$, which is produced by electrons and acts upon the lattice:

$$F(x,t)=\frac{\partial}{\partial x}\Phi(x,t)\,,$$

$$\Phi(x,t)=\int\frac{2\,\mathrm{d}\boldsymbol{p}}{(2\pi\hbar)^3}\Lambda_{xx}(\boldsymbol{p})\frac{\partial f_\mathrm{F}}{\partial\epsilon}\chi\,, \tag{9}$$

where $(\partial f_\mathrm{F}/\partial\epsilon)\chi$ is the nonadiabatic addition to the Fermi distribution function f_F, $\boldsymbol{p}$ is the electron momentum. In the model of the electronic dispersion law that we use, the component $\Lambda_{xx}(\boldsymbol{p})$ of the deformation potential has the form

$$\Lambda_{xx}(\boldsymbol{p})=-\tilde{m}(v_x^2-\tfrac{1}{3}v_\mathrm{F}^2)\,, \tag{10}$$

where $\tilde{m}$ is the "deformation" mass, $\boldsymbol{v}=\boldsymbol{p}/m$ the electron velocity, v_F the Fermi velocity.

In the course of the derivation of (7) and (8) we have used the accurate boundary condition which considers the deformation contribution [18,19]:

$$[\rho_0 s^2\,\mathrm{d}u(x,t)/\mathrm{d}x+\Phi(x,t)]\,|_{x=0}=0\,. \tag{11}$$

In (7) and (8) it was also considered that the attenuation sound length l_s is the largest parameter in the problem ($l_s\gg\delta$, $l_s\sim v_\mathrm{F}/sq\gg 1/q$ and $l_s\sim l\nu/\omega\gg l$).

Let us emphasize that formula (8) was obtained for the case when the deformation force gives the fundamental contribution to u_n, and consequently to $u(x,t)$. The analysis shows that the induction contribution to $u(x,t)$ is negligible in comparison with the deformation one when the inequality

$$(ql/b)^2\,\mathrm{th}(b^2)\gg 1 \tag{12}$$

is satisfied.

3. In the present work with the aid of a detailed analysis of the conduction-electrons' dynamics in a heterogeneous magnetic field $H(x,t)+h_0$, the nonlinear kinetic equation was solved, and we have found χ. A general expression for the deformation force has been obtained. On the basis of this expression we examine the cases of weak ($b\ll 1$) and developed ($b\gtrsim 1$) nonlinearities.

In conditions of weak nonlinearity and a small magnitude for the external field h_0:

$$h_0,\,2\mathscr{H}\ll \mathscr{h}\,, \tag{13}$$

the magnetic field slightly curves the electron trajectories in the skin layer, and in leading order approximations in $h_0/\mathscr{h}$ and b^2, the first ($n=\pm 1$) and second ($n=\pm 2$) acoustic harmonics are generated. Other harmonics ($n=\pm 3$, ± 4, ...) are of higher orders in the small parameters $h_0/\mathscr{h}$ and b^2. The amplitude of the first harmonic ($n=1$) has the form

$$u_1=-\frac{\mathrm{i}}{12\pi^2}\frac{\tilde{m}}{m}\rho\frac{h_0\mathscr{H}}{\rho_0\omega s}\left(\frac{l}{\delta_\mathrm{a}}\right)^2 G(q\delta_\mathrm{a})\,,$$

$$G(q\delta_\mathrm{a})=\int\limits_{\substack{c-\mathrm{i}\infty\\-1<c=\mathrm{Re}\,z<1-z_0}}^{c+\mathrm{i}\infty}\mathrm{d}z\,M(z)(q\delta_\mathrm{a})^{z+1}z(z+1)$$

$$\times\frac{\sin^2(\pi z/2)}{\sin^2(\pi z)}\,. \tag{14}$$

Here ρ represents the probability of specular reflection of electrons from the boundary, $z_0=(\arccos\rho)/\pi$, δ_a is the skin thickness of linear theory,

$$\delta_\mathrm{a}=(c^2l/3\pi^2\sigma_0\omega)^{1/3}\,, \tag{15}$$

τ_0 is the massive-metal static conductivity. The explicit form of the function $M(z)$ is exhibited in refs. [20,21].

In leading order approximation in b^2 the second amplitude u_2 coincides with the result obtained for $h_0=0$. This amplitude was calculated in refs. [16,17].

From the result (14) it follows that the first harmonic of the longitudinal sound is excited because of the presence of the external constant magnetic field h_0. Expression (14) for u_1, valid under conditions (13), coincides with the result found in the linear theory when $b=0$. In contrast with the excitation of the first harmonic, the generation of the second harmonic is due to the heterogeneity of the magnetic wave field $H(x,t)$, and it is not connected with the presence of the external field h_0. Other acoustic harmonics (3ω, 4ω, ...) appear owing to the interference of the first and second harmonics.

4. In the region of external magnetic fields $|h_0|\leqslant 2\mathscr{H}$, where current states are excited, the regime of developed nonlinearity ($b\gtrsim 1$) is characterized by the fact that some of the electrons are trapped by the resultant heterogeneous magnetic field $H(x,t)+h_0$ in the time intervals, when

$$\frac{2\mathscr{H}\cos(\omega t)+h_0}{h+h_0}<0\,. \tag{16}$$

During this time the spatial distribution of the field $H(x, t)+h_0$ is variable of sign, and, precisely for this reason, in the metal a group of trapped electrons appears with trajectories that wind around the plane where the magnetic field changes its sign. The periodic appearance and disappearance of the group of trapped electrons causes the jump-like time-dependence of the metal conductivity and, as a consequence, the effect of current rectification.

Under conditions of current states the harmonic amplitudes u_n in (7) are of the same order of magnitude. Considering that at low temperatures under conditions of the quasistatic ($\omega \ll \nu$) anomalous skin effect the case $q\delta \ll 1$ ($\omega \ll s/\delta$) is of great interest, we will show the asymptotically exact expression for u_n in the situation $b \gg 1$, $q\delta \ll 1$ and $R \gg l$:

$$u_n = \frac{4\pi\sqrt{3}}{9} \bar{U}(-1)^{n\theta(\bar{\kappa})} \sin(\beta) \sin(n\beta) \mu^{4/3} \times \sum_{r=1}^{\infty} \frac{n\mu r + i\sqrt{3}[r^2 - 2(n\mu)^2]}{r^{1/3}(r^2-\mu^2)[r^2-(n\mu)^2]} . \quad (17)$$

Here we have introduced the following notation:

$$\bar{U} = \frac{4}{3\pi^5} \frac{\bar{m}}{m} \frac{\mathscr{H}^2}{p_0 \omega s} \frac{R}{\delta_a} (q\delta_a)^2 , \quad (18)$$

$$\mu = (\pi - \beta)/\pi , \quad \beta = \arccos[(h_0/2\mathscr{H}) \operatorname{sign}(\bar{\kappa})] , \quad (19)$$

where $\theta(\bar{\kappa})$ is the Heaviside function, $\bar{\kappa} \equiv (h_0 + h)/2\mathscr{H}$.

Let us remark that with $b \gg 1$ the induced field h and, therefore, $\operatorname{sign}(\bar{\kappa})$ in (17), (19) ($|\bar{\kappa}| \leqslant 1$), are double-valued functions of the field h_0 [9,10,13]. Hence, the found amplitude u_n (17) also turns out to be a double-valued function of h_0. Graphs of the amplitude $|u_n|$ ($n=1, 2, 3$) are shown in figs. 1–6 (see $b \to \infty$).

It is interesting to note that the number of terms in (17), which determines the fundamental dependence of u_n on h_0, is equal to $|n|$ ($r \leqslant n$).

Result (17), as well as the results of sections 5 and 6, is referred to as the diffuse reflection of electrons from the metal boundary ($\rho = 0$).

5. The comparison of the results obtained in sections 3 and 4, shows that in the case of weak nonlinearity ($b \ll 1$) the dependence of the amplitudes

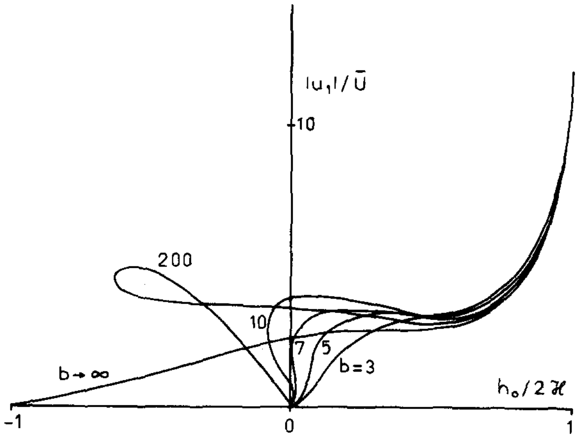

Fig. 1. Computational results for the amplitudes of the first harmonic ($|u_1|$) with $\operatorname{sign}(\bar{\kappa})>0$ and different values for parameter b.

u_n on h_0 is single-valued, and in the regime of strong nonlinearity ($b \gg 1$) it is non-single-valued. Subsequently, there is a critical value $b_c \sim 1$ with a corresponding amplitude value $\mathscr{H}_c = k/2b_c^2$, at which that non-single-valued nature appears for the first time. In this section we will study the appearance and development of the non-single-valued dependence of u_n on h_0.

On the basis of the found asymptotically exact expressions for the deformation force in conditions of weak ($b \ll 1$) and strong ($b \gg 1$) nonlinearities, we have constructed a model for this force, which is valid for arbitrary b. With its aid we were able to in-

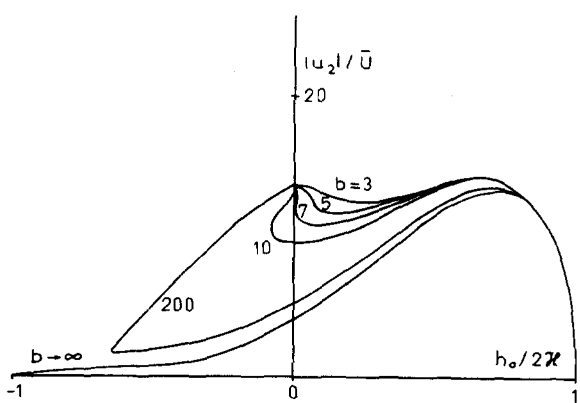

Fig. 2. Computational results for the amplitudes of the second harmonic ($|u_2|$) with $\operatorname{sign}(\bar{\kappa})>0$ and different values for parameter b.

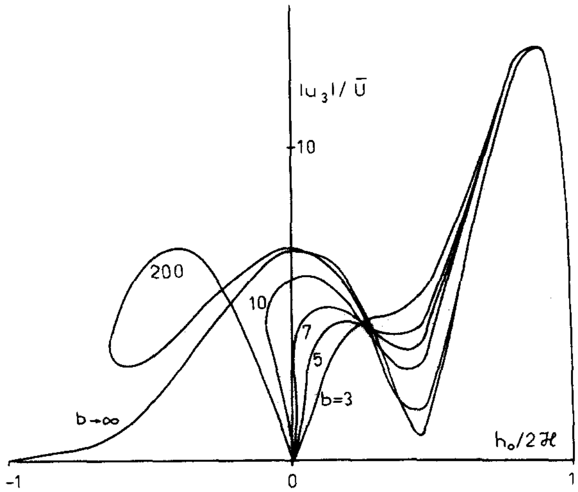

Fig. 3. Computational results for the amplitudes of the third harmonic ($|u_3|$) with sign($\bar{\kappa}$) > 0 and different values for parameter b.

vestigate the sound generation in the region of amplitudes $\mathscr{H}$, where the current states originated. According to this model, the expression for amplitudes u_n (8) with $q\delta \ll 1$ and $R \gg l$ has the form

$$u_n = \mathrm{i}n(-1)^{n\theta(\bar{\kappa})} \frac{2\pi\sqrt{3}}{9} \bar{U} \frac{\alpha-1}{\alpha\mu^{2/3}}$$
$$\times \sum_{\substack{r=-\infty \\ r\neq 0}}^{\infty} \frac{\exp[\mathrm{i}\pi\,\mathrm{sign}(r)/6]}{|r|^{1/3}} \left(\frac{\sin[(1+r/\mu\alpha)\beta]}{(1+r/\mu\alpha)(1+r/\mu)} + \frac{\sin[(1-r/\mu\alpha)\beta]}{(1-r/\mu\alpha)(1-r/\mu)} \right)$$
$$\times \left(\int_0^{\beta} \mathrm{d}\varphi\, \mathrm{th}\{b^2[h_0\,\mathrm{sign}(\bar{\kappa})/2\mathscr{H} - \cos(\varphi)]\} \times \cos[(n-r/\mu\alpha)\varphi] + (-1)^{n+r} \int_0^{\pi-\beta} \mathrm{d}\varphi\, \mathrm{th}\{b^2[h_0\,\mathrm{sign}(\bar{\kappa})/2\mathscr{H} + \cos(\varphi)]\} \cos[(n-r/\mu)\varphi] \right). \quad (20)$$

In (20) have we used

$$\alpha = \frac{1}{1-\exp(1/b|\bar{\kappa}|)}, \quad \mu = \frac{\pi-\beta+\beta/\alpha}{\pi}. \quad (21)$$

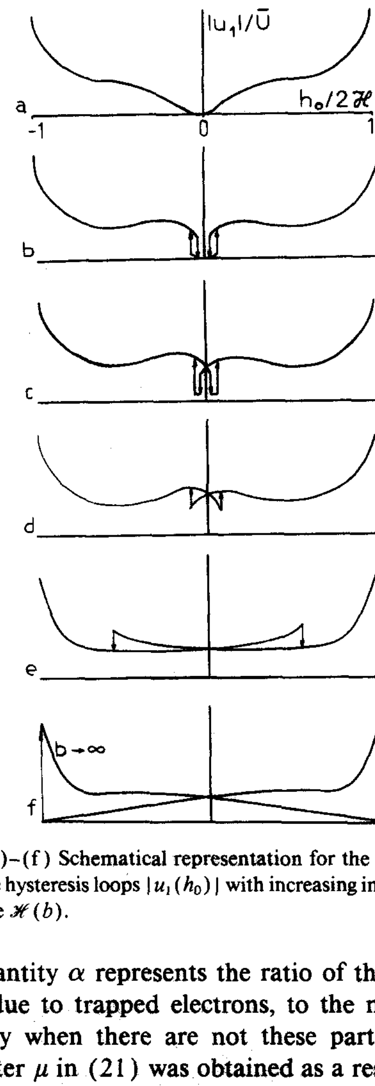

Fig. 4. (a)–(f) Schematical representation for the shape variation of the hysteresis loops $|u_1(h_0)|$ with increasing incident-wave amplitude $\mathscr{H}(b)$.

The quantity α represents the ratio of the conductivity, due to trapped electrons, to the metal conductivity when there are not these particles. The parameter μ in (21) was obtained as a result of the time averaging of the metal resistance divided by its value when the field $H(x, t) + h_0$ is sign-constant.

The induced field, present in $\bar{\kappa}$ ($\bar{\kappa} = (h_0 + h)/2\mathscr{H}$), is obtained by resolving the equation [13]

$$h/2\mathscr{H} = \{[1-(h_0/2\mathscr{H})^2]^{1/2}\,\mathrm{sign}(\bar{\kappa})\} \times \{\arccos[-(h_0/2\mathscr{H})\,\mathrm{sign}(\bar{\kappa})] + \pi[\exp(1/b|\bar{\kappa}|)-1]\}^{-1}. \quad (22)$$

The parameter b grows as the amplitude $\mathscr{H}$ is increased, and when $b = b_c$ the dependence of h on h_0, determined by relation (22), becomes non-single-valued. In the region $b > b_c$ this dependence has a hysteresis behavior [9,10,13]. It is natural to expect

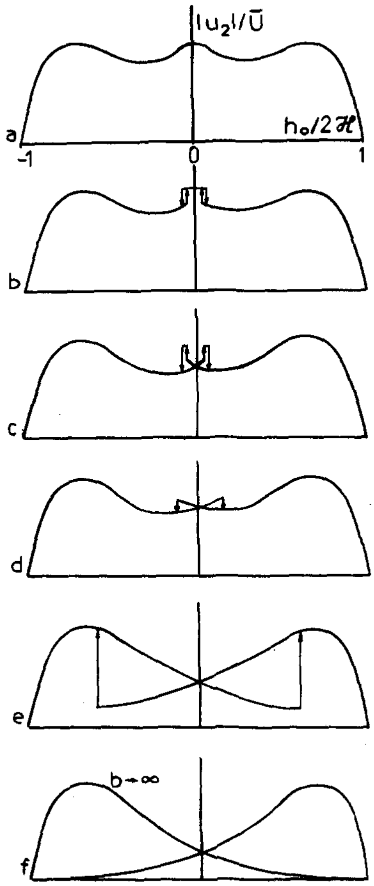

Fig. 5. (a)–(f) Schematical representation for the shape variation of the hysteresis loops $|u_2(h_0)|$ with increasing incident-wave amplitude $\mathscr{H}(b)$.

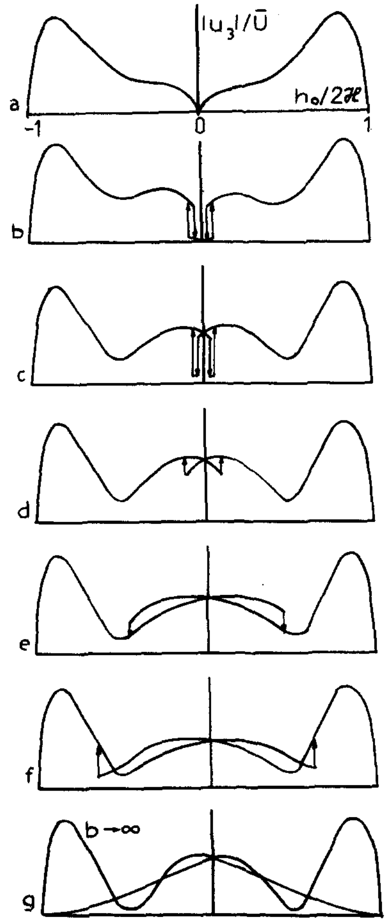

Fig. 6. (a)–(g) Schematical representation for the shape variation of the hysteresis loops $|u_3(h_0)|$ with increasing incident-wave amplitude $\mathscr{H}(b)$.

that the hysteresis dependence of the induced field h on the external field h_0 has to produce the hysteresis of the acoustic harmonics amplitudes u_n (20), the magnitudes of which depend on the magnetic fields h and h_0.

The computational results for the dependence of the first-, second-, and third-harmonic amplitudes of the generated longitudinal sound on the external constant magnetic field h_0, for distinct values of the nonlinearity parameter b (of amplitudes $\mathscr{H}$), are shown in figs. 1–3. With sufficiently small amplitudes $\mathscr{H}$, when the parameter b is smaller than a critical value $b_c \sim 5$, the dependence $|u_1(h_0)|$, $|u_2(h_0)|$, $|u_3(h_0)|$ are single-valued. At $b=b_c$, points with vertical tangents appear in the graphs of $|u_1(h_0)|$, $|u_2(h_0)|$, $|u_3(h_0)|$, and with $b>b_c$ these amplitudes become non-single-valued functions of the external magnetic field. The instability of the solutions $|u_n(h_0)|$ $(n=1, 2, 3)$ on the intervals enclosed between points h_0^* where the derivative $\partial|u_n(h_0^*)|/\partial h_0$ tends to infinity, causes amplitude jumps at $h_0=h_0^*$. Therefore, beginning at the value $b=b_c$ $(\mathscr{H}=\mathscr{H}_c)$, the dependence of amplitudes $|u_n|$ on h_0 acquires a hysteresis character. Sets of hysteresis curves $|u_n(h_0)|$ $(n=1, 2, 3)$ are schematically drawn in figs. 4–6. Naturally, the quantities b_c and h_0^* for amplitudes $|u_n(h_0)|$ coincide with the respective values for the dependence of h on h_0.

The hysteresis loops for amplitudes $|u_n|$, $n=1, 2, 3$, are appreciably distinct from each other. This is evident with $|h_0/2\mathscr{H}|\to 1$, when the resultant field

$H(x, t)+h_0$ becomes sign-constant at each moment and the trapped electrons disappear. Weakening the nonlinearity, the amplitudes $|u_2|$, $|u_3|$ decrease in comparison with $|u_1|$. At $h_0=0$ the amplitudes of the first and third harmonic vanish. This is related with the fact that in the absence of an external constant magnetic field, only the even harmonics (2ω, 4ω, ...) are generated [16,17]. Note that hysteresis loops of amplitudes $|u_n|$ for $|n|>3$ are similar to loops of the amplitude for the third harmonic $|u_3(h_0)|$.

6. Finally, we will exhibit one more result which, in our opinion, is of great interest. It turns out that in conditions of developed nonlinearity and with $q\delta \ll 1$, the sound field $u(x, t)$ (7) in the region $x \gg l$ is determined by the behavior of the electric field $E(0, t)$ on the metal surface $x=0$:

$$u(x, t) = -\frac{1}{3\pi^2}\frac{\tilde{m}}{m}\frac{c^2 p_F}{p_0 s^3 e}$$
$$\times \operatorname{sign}[2\mathscr{H}\cos(\omega t - qx)+h_0]E(0, t')+u_0 ,$$
$$t' = t - x/s . \tag{23}$$

Note that in the dependence (23) for the sound field there are also delta-function spikes, which correspond to times when the resultant magnetic field on the sample boundary vanishes.

In refs. [11,12] it has been shown theoretically and experimentally that the electric field $E(0, t)$ experiences jump-like alterations due to conductivity jumps at the moment of appearance and disappearance of trapped particles in the sample. Under conditions (16) when the resultant magnetic field in the sample is sign-variable, the amplitude $E(0, t')$ is $(1/\alpha)^{1/3} \ll 1$ times smaller than the characteristic amplitude of the electric field at times, when there are no trapped electrons. From (22) it follows that jumps, similar to jumps of $E(0, t')$, should be observed in the total displacement $u(x, t) \equiv u(\varphi)$ ($\varphi = \omega t - qx$) within each sound-wave period ($u(\varphi+2\pi) = u(\varphi)$). Since the argument of displacement $u(\varphi) \equiv u(\omega t - qx)$ contains the "retardation" parameter qx, the jumps of amplitude $u(\varphi)$ occur at times x/s later than jumps of the electric field $E(0, t')$.

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