Physica C 197 (1992) 161-166
North-Holland

PHYSICA C

Non-linear interaction of a transport current with an electromagnetic wave in high-T_c ceramics

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Received 12 September 1991

We have studied theoretically and experimentally a non-linear interaction of a radio wave with a transport current in high-T_c ceramics in the critical state. It was established that for currents I lower than the critical value I_c the dependence of the surface impedance of a sample on a transport current is connected with the dependence of the critical current density on the self-magnetic field of a current. When a sample transits to the resistive state and the longitudinal electric field appears, the mentioned interaction leads to sharp decreasing of screening currents. Accordingly, the AC field penetration depth increases. The real part of the surface resistance, as a function of the current I, has a maximum in the region near $I \approx I_c$. If the frequency of a radio wave increases, the position of this maximum is displaced to the region of higher currents. We should like to note that it is impossible to explain an observed phenomenon if one considers ceramic media in the resistive state on the basis of the usual skin effect in normal metal.

1. Introduction

The investigation of the electromagnetic properties of high-T_c samples permits one to obtain important information about different intrinsic characteristics of these materials. For example, using the measurements of the surface impedance Z of ceramic plates of cylinders, we have developed a new contactless method for the determination of the magnetic field dependence of the critical current density $J_c(B)$ [1]. Some of us have shown that the measurement of the surface impedance dependence on a transport current I is an effective tool for finding the value of the critical current I_c with high accuracy [2]. When the transport current reaches the critical value I_c and a sample transits to the resistive state, the surface resistance $\mathcal{R}$ begins to increase sharply to its maximum value (see fig. 1 of paper [2]).

Similar maxima in the dependences of the surface resistance on different parameters such as temperature, external magnetic field, frequency etc. have been observed in a number of papers beginning from 1987. The physical nature of these maxima is considered as well established. The simplest explanation can be obtained on the basis of the old paper [3]. When one of the mentioned parameters is changed, the AC magnetic field penetration depth δ changes too. At some value of the external parameter, the penetration depth δ proves to be equal to the thickness d of a superconducting plate or to the radius R of a cylinder sample. According to ref. [3] an electromagnetic absorption has a maximum in this case.

One can think that the nature of the maximum in this fig. 1 can be easily explained in the same way. As a matter of fact, while a sample is in the superconducting state ($I<I_c$) and AC magnetic field amplitude H_m is small, the AC penetration depth and electromagnetic losses are very small too. When current I exceeds I_c, the sample is in the resistive state and this system, from the first point of view, reminds one of a normal metal having bad conductivity. In this case, the AC penetration depth may be much greater than the sample size and the losses are again

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small. Naturally, δ may compare with the sample size in the intermediate region $I \simeq I_c$ and the surface resistance $\mathscr{R}$ must have a maximum here.

A simple but more careful consideration shows that this point of view is superficial.We can evaluate the effective conductivity σ of our sample in the region of a maximum of $\mathscr{R}(I)$ using the usual V–I plot presented in fig. 1. The result of such a calculation gives $\sigma > 10^{20}$ s^{-1}. Using the formulae for δ in the case of a normal skin effect, we obtain that the penetration depth proves to be much smaller than 1 mm while the diameter of our samples was 10 mm. Thus, we can see that the simple consideration of superconducting ceramics in the resistive state as a metal medium is invalid. We can conclude that the real nature of a maximum of $\mathscr{R}(I)$ is connected with the nonlinear interaction of a transport current with an electromagnetic wave in a ceramic sample.

This paper is devoted to the description of such an interaction. Below, we shall show that the observed phenomenon is the result of the specific properties of a superconducting ceramic medium described by the critical state model. In the framework of this model, the absolute value of the current density is equal to the critical value J_c independently of the electric field E. When both the transport current I and and AC field exist in a sample, the full current in ceramics contains two components (longitudinal component J_z and screening one J_{scr}). As was shown in refs. [4,5], these components are flowing in separate parts of a sample and so the absolute values of each of them are equal to J_c. The transition of a sample to the resistive state changes the situation radically and leads to the mixing of both these currents in the full region where AC field exists. The direction of a total current $\boldsymbol{J}$ is parallel to the vector of electric field $\boldsymbol{E}$: $\boldsymbol{J}=J_c \cdot \boldsymbol{E}/E$. So, at large E_z a total current is oriented almost along the sample z-axis. For this reason a sharp decreasing of screening currents and an increasing of the penetration depth take place. So, as a result of such specific interaction between the transport current and AC field, the penetration depth proves to be much higher than one calculated by the simple formula for the usual skin effect.

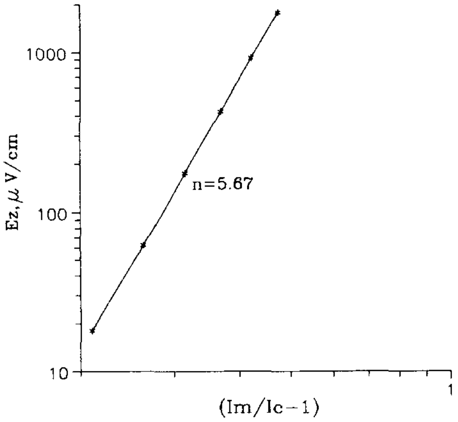

Fig. 1.The V–I characteristic of the sample in a double logarithmic scale.

2. Theoretical

It is well known that the electrodynamics of high-T_c superconductors may be described by the critical state model [6,7] in a wide range of magnetic fields as well as of amplitudes and frequencies of electromagnetic waves. In particular, the equation describing the distribution of the magnetic induction $\boldsymbol{B}$ inside a ceramic medium has the form [7,4]

$$\operatorname{curl}\boldsymbol{B} = (4\pi/c)\cdot\mu J_c(B)\cdot\boldsymbol{E}/E\,. \tag{1}$$

Here, $\boldsymbol{E}$ is the electric field, μ is the effective magnetic permeability of ceramics where only the intragranular currents are taken into account; $J_c(B)$ is the critical intergranular current density depending on the magnetic induction; and c is the speed of light.

Let us consider a high-T_c ceramic sample having a cylindrical form with radius R which is placed in an external magnetic field having the form:

$$H(t) = H_m \cos\omega t\,. \tag{2}$$

Let the transport current I flow through the cylinder along its z-axis. We shall consider the case of a small amplitude H_m:

$$H_m \ll B^*/\mu\,, \tag{3}$$

where B^* is the magnetic induction scale where the critical current density J_c decreases noticeably. According to eq. (3), the critical current density $J_c(B)$ in eq. (1) depends only on the self-magnetic field of a transport current. Besides, we shall suppose that the next inequality is fulfilled:

$$H_{\mathrm{m}} \ll 4\pi R J_{\mathrm{c}}/c\,. \tag{4}$$

In this case, the AC magnetic field penetration depth $\Delta \approx cH_{\mathrm{m}}/4\pi J_{\mathrm{c}}$ is less than the sample radius R in a full range of critical current changing from zero to I_{c}. In the examined situation, the self-magnetic induction B of the current I does not differ noticeably from its value $B_\varphi(R)$ on the sample surface in the whole region of a sample where the AC magnetic field exists:

$$B \approx B_\varphi(R) = 2\mu I/Rc\,. \tag{5}$$

So, the critical current density J_{c} in eq. (1) is defined by the full transport current I:

$$J_{\mathrm{c}}(B) = J_{\mathrm{c}}(B_\varphi(R)) = J_{\mathrm{c}}(I)\,. \tag{6}$$

Furthermore, to investigate the interaction between the transport current and the external radio wave, we shall consider the dependence of the surface impedance of sample on the transport current I as a physical parameter characterizing this interaction. The surface impedance Z is defined in our geometry by the relations:

$$Z = \mathcal{R} - \mathrm{i}\mathcal{X} = (8\pi/c)E_\omega(R)/H_{\mathrm{m}}\,,$$

$$E_\omega = (\omega/2\pi)\int_0^{2\pi/\omega} E(t)\exp(\mathrm{i}\omega t)\,\mathrm{d}t\,, \tag{7}$$

where $\mathcal{R}$ and $\mathcal{X}$ are the surface resistance and the surface reactance, respectively.

If the transport current I is less than I_{c}, only the azimuthal component of the electric field exists. So, the critical state eq. (1) may be re-written in the simple form

$$\partial B_z/\partial r = -(4\pi/c)\mu J_{\mathrm{c}}(I)\,\mathrm{sign}\,E_\varphi\,, \tag{8}$$

where r is the radial coordinate. The second Maxwell's equation, which connects the fields E_φ and B_z, has the form

$$\partial E_\varphi/\partial r + E_\varphi/r = -(1/c)\partial B_z/\partial t. \tag{9}$$

The boundary conditions for eqs. (8) and (9) have the form:

$$B_z(R,t) = \mu H_{\mathrm{m}}\cos\omega t, \qquad E_\varphi(0,t) = 0\,. \tag{10}$$

Omitting the simple mathematical transformations, we give here the final expression for the surface impedance:

$$Z = (2/3\pi)[\mu\omega H_{\mathrm{m}}/cJ_{\mathrm{c}}(I)] \times (1 - 3\pi\mathrm{I}/4),\; I < I_{\mathrm{c}}\,. \tag{11}$$

We can see that the surface impedance is a function of H_{m} even for small values of the AC magnetic field amplitude. This result is connected with a linear increase of the AC field penetration depth Δ into a sample when H_{m} increases and, as a consequence, with a squared increase of the electric field $E_\varphi(R,t)$. The impedance phase multiplier in eq. (11) is also unusual.

Let us note that the result (11) is analogous to the expression for the surface impedance of a sample placed in an external DC magnetic field H_0 in the absence of a transport current [8,1]. The difference consists in the substitution of the external DC magnetic field H_0 with the self-magnetic field $B_\varphi(R)$ of a transport current. In connection with this conclusion, the function $J_{\mathrm{c}}(I)$ appears in the denominator of relation (11). Earlier [5,1], a new contactless method was developed to determine the local critical current density in ceramics and its dependence on B. This method is based on the obvious connection of the surface impedance and the function $J_{\mathrm{c}}(B)$. It is clear that this function may be re-established, using relations (11) and (6) and the results of the measurements of the surface impedance, as a function of the transport current I.

Let us now consider the more interesting situation when the transport current is higher than the critical one. In this case, as is well known, a sample transits to the resistive state, i.e., a longitudinal electric field E_z, parallel to the transport current, appears. Until the electric field E_z is small and does not exceed the characteristic value of azimuthal electric field E_φ,

$$E_z < E_\varphi \sim \mu H_{\mathrm{m}}^2\omega/[8\pi J_{\mathrm{c}}(I)]\,, \tag{12}$$

the result (11) remains qualitatively correct. In the case

$$E_z \gg \mu H_{\mathrm{m}}^2\omega/[8\pi J_{\mathrm{c}}(I)]\,, \tag{13}$$

the picture of AC magnetic field penetration into a sample changes radically. As we can see from eq. (1), the significant decrease of the azimuthal component of the current density occurs at the condition (13):

$$J_\varphi \approx J_{\mathrm{c}}(I_{\mathrm{c}})E_\varphi/E \ll J_{\mathrm{c}}(I_{\mathrm{c}})\,. \tag{14}$$

This means that the transition of a high-T_c sample to the resistive state leads to an abrupt drop of the screening current J_φ: the direction of the vector of the current density $\boldsymbol{J}_c = J_c\boldsymbol{E}/E$ proved to be approximately parallel to the z-axis. Maxwell equations (eqs. (15) and (9)) for an AC field at the condition (13) acquire the form typical for the normal skin effect:

$$\partial B_z/\partial r = -(4\pi/c)\sigma_{\mathrm{eff}}E_\varphi\,, \tag{15}$$

where σ_{eff} is the effective conductivity depending on the value of the transport current I,

$$\sigma_{\mathrm{eff}} = \mu J_c(I_c)/E_z\,. \tag{16}$$

The system of eqs. (15) and (9) is a linear system with respect to the fields B_z and E_φ. Its solution, taking into account the boundary conditions (10), gives

$$B_z(r,t) = \mathrm{Re}[B(r)\exp(-\mathrm{i}\omega t)]\,,$$

$$B(r) = \mu H_m[J_0(\mathrm{i}r/\delta)/J_0(\mathrm{i}R/\delta)]\,, \tag{17}$$

$$\delta = \delta_n(1+\mathrm{i})/2, \quad \delta_n^2 = c^2/(2\pi\sigma_{\mathrm{eff}}\omega)\,. \tag{18}$$

Here, δ_n is the effective skin-depth, J_0 is the Bessel function. The expression for the surface impedance has the form:

$$Z = (4\pi\mu\delta\omega/c^2)[J_1(\mathrm{i}R/\delta)/J_0(\mathrm{i}R/\delta)],$$

$$I > I_c\,. \tag{19}$$

On the basis of eqs. (16) and (18), we can see that the effective conductivity σ_{eff} decreases quickly and the skin-depth δ_n increases when the electric field E_z grows. Due to quick increasing of δ_n, the argument of the Bessel functions in eqs. (17) and (19) changes from a value higher than unity to a small quantity in comparison to unity. If the electric field E_z is not very high and satisfies the inequalities

$$\mu H_m^2\omega/[8\pi J_c(I_c)] \ll E_z \ll 4\pi\mu\omega J_c(I_c)R^2/c\,, \tag{20}$$

the skin-depth δ_n (18) proves to be much smaller than the sample radius R. In this case, we can use the asymptotic expansion of the Bessel function for large arguments. As a result, instead of relations (16), (17) and (19), we obtain the following expressions for the field $B(r)$ and for the surface impedance Z in the case $\delta_n \ll R$:

$$B(r) = \mu H_m(R/r)^{1/2}\exp[-(R-r)/\delta],$$

$$\delta_n \ll r, R\,, \tag{21}$$

$$Z = -4\pi\mu\omega\delta/c^2 = (\mu/c)(1-\mathrm{i})(2\pi\omega/\sigma_{\mathrm{eff}})^{1/2}$$

$$= [2\pi\mu\omega/J_c(I)c^2]^{1/2}(1-\mathrm{i})E_z^{1/2}\,. \tag{22}$$

The surface impedance increases quickly when a transport current I and, therefore, the electric field $E_{\mathrm{rm}z}$ increase abruptly. During this process, the real and imaginary parts of the surface impedance prove to be equal to each other. If the field E_z increases, the penetration depth of the AC field increases also, and at some value of E_z the inequality (20) is changed to an opposite one. In the case when

$$E_z \gg 4\pi\mu\omega J_c(I)R^2/c^2, \text{ or } R \ll \delta_n\,, \tag{23}$$

the AC field penetrates into the whole sample. In such a situation the asymptotic expression for the surface impedance at small values of parameter R/δ_n has the form:

$$Z = -2\pi\mathrm{i}\mu\omega R(1+\mathrm{i}R^2/4\delta_n^2)/c^2$$

$$= -2\pi\mathrm{i}\mu\omega R/c^2 + \pi\mu\omega^2R^3J_c(I)/c^4\,. \tag{24}$$

The imaginary part of the impedance $\mathscr{X}$ reaches a maximum value

$$\mathscr{X}_{\max} = 2\pi\mu\omega R/c^2\,, \tag{25}$$

but the real part decreases when E_z grows,

$$\mathscr{R} = \mathrm{Re}\,Z \propto E_z^{-1}\,. \tag{26}$$

So, owing to the dependence of the critical current density on magnetic field, the surface impedance of a ceramic sample increases slowly when the transport current rises from zero to I_c. Increasing of the transport current is accompanied by decreasing of the screening current J_φ that leads to the growth of the penetration depth of the AC field and, accordingly, of the surface impedance. When a sample transits to the resistive state ($I > I_c$), the screening currents decrease abruptly owing to the appearance of a longitudinal electric field E_z. In this region of current I, the surface impedance is described by the formulae of the normal skin effect. At first, when $\delta_n \ll R$, the real and imaginary parts of the surface impedance are equal to each other and are an increase function of E_z. Then, in the region $\delta_n \gg R$, the real part of the impedance begins to decrease but the imaginary part tends to its limited value $\mathscr{X}_{\max}$ (25). This means that, in the intermediate region,

$$\delta_n \approx R, \quad \text{or} \quad E_z \approx 4\pi\mu\omega J_c R^2/c^2 \tag{27}$$

and the real part of the impedance has a maximum the height of which may be evaluated by the formula:

$$\mathcal{R}_{\max} \approx 2\pi\mu\omega R/c^2 . \tag{28}$$

The nature of this maximum is analogous to the effect of Fisher and Kao [3], well known in the theory of the normal skin effect in normal metals. Increasing of the impedance at $\delta_n < R$ is connected with the growth of the AC field penetration depth into a sample. Diminishing of the surface impedance at $\delta_n > R$ is conditioned by the transparency of the sample, and the AC electric field component which is in phase with the AC magnetic field effectively decreases. It needs to be noted that, owing to the fast growth of a longitudinal electric field E_z with increasing I in the resistive region, the maximum of $\mathcal{R}(I)$ proves to be sharp enough.

3. Experimental

Figure 2 shows the experimental dependence of the real and imaginary parts of the surface impedance on transport current for the sample of yttrium ceramics having size $5 \times 6 \times 7$ mm^3. These results were obtained at an amplitude of the external magnetic field of 2 Oe and at a frequency of 1 kHz. The form of the curves $\mathcal{R}(I)$ and $\mathcal{X}(I)$ corresponds completely to the theoretical analysis presented above. A slow growth of the impedance in the region $I \ll I_c$ is changed by the abrupt dependence in the resistive region. The value of I_c for this sample is equal to 20 A.

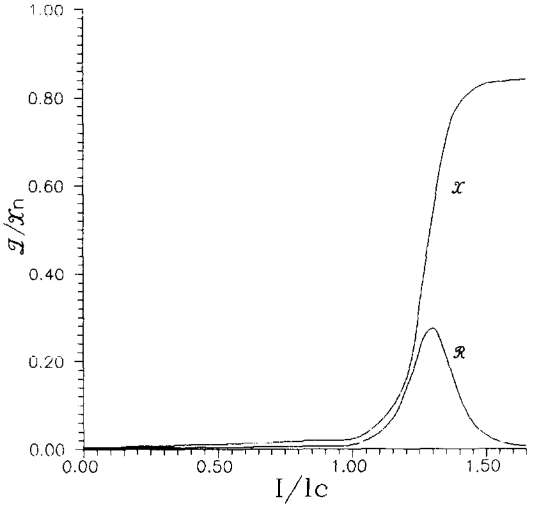

Fig. 2. The dependences of the real $\mathcal{R}$ and imaginary ($\mathcal{X}$) parts of the surface impedance on a current I, $\omega/2\pi = 1000$ Hz, $H_m = 1$ Oe.

The position of the maximum of $\mathcal{R}(I)$ depends on the frequency of the AC field: increasing the frequency leads to the displacement of this maximum to higher currents (see fig. 2). This fact obtains a simple explanation in the framework of the considerations given above. As a matter of fact, according to relation (27), defining the position of the maximum as a function of the electric field E_z, we may find that at higher frequency the maximum must be observed at the higher E_z and accordingly a higher current I. As follows from the analysis of V–I plots in the region $I > I_c$, the field E_z is described by the power dependence on the difference $I - I_c$ (see fig. 3):

$$E_z \approx (I - I_c)^{\alpha}, \quad \alpha = 5.67 . \tag{29}$$

This means that the position $I_{\max}$ of the maximum of $\mathcal{R}(I)$ must depend on frequency as;

$$I_{\max} - I_c \propto \omega^{1/\alpha} . \tag{30}$$

This result is in good agreement with the experimental data (see fig. 2).

It needs to be noted that the employment of our

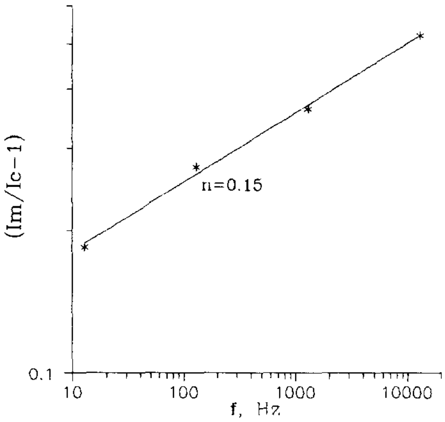

Fig. 3. The dependence of the maximum position $I_{\max}$ of the function $\mathcal{R}(I)$ on frequency in a double logarithmic scale.

model of the critical state is invalid at high frequencies and the position of the maximum of $\mathcal{R}(I)$ is displaced to the region of higher currents. At high currents one needs to add an additional item connected with the "supercritical" state:

$$\operatorname{curl} \boldsymbol{B} = (4\pi\mu/c)\,[J_c(B) + AE^{1/\alpha}]\cdot \boldsymbol{E}/E\,, \tag{31}$$

where $A = \text{const}$. We wrote the formula (31) in accordance with the character of the V–I plot discussed above (see (29) and fig. 1).

Using (31), we can improve the relation for the effective conductivity σ_{eff}:

$$\sigma_{\text{eff}} = \mu J_c(I)/E_z + \mu A/E_z(\alpha-1)/\alpha\,. \tag{32}$$

When E_z is not very high, i.e., I is near I_c, the main item in the sum (32) is the first one. However, this item decreases more quickly with E_z than the second item. As a result, we have competition between two different mechanisms of forming an effective conductivity σ_{eff}. So, at high frequencies, relation (21) becomes invalid for the description of the maximum position. Instead of this relation, the next correlation becomes correct:

$$I_{\max} - I_c \propto \omega^{1/(\alpha-1)}\,. \tag{33}$$

This means that the displacement of the maximum position, as a function of ω, grows faster than at low frequencies.

Acknowledgement

This work is supported by the Science Council on high-T_c superconductivity, project 90067.

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