Size effect in hard superconductors at unilateral excitation

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It is shown that size effect plays an important role in the ac response of hard superconductors not only at bilateral but at unilateral excitation as well. This effect manifests itself in the ac amplitude dependences of the surface impedance and absorptivity. We find that the absorptivity of a superconductor has always a prominent maximum independently of the dielectric properties of the substrate. This statement is demonstrated by theoretical calculations performed within the critical state model.

The size effect is well known in the electrodynamics of metals[1,2] and superconductors.[3–8] This effect has not only academic interest but also plays an important role in applications. For example, it is used as a basis of the new contactless method for testing critical parameters of superconductors.[8]

The size effect at bilateral excitation manifests itself as a steplike change of the imaginary part X of the rf surface impedance $Z=R-iX$ (of the real part of dynamic magnetic susceptibility) and as a peak in R (in the imaginary part of magnetic susceptibility) versus ac frequency, temperature, external magnetic field and other parameters of the problem. Such peculiarities of the surface impedance were observed, for instance, in Refs. 9–11. It is noteworthy that some authors tried to explain experimental results in terms of the inverse relaxation time of the vortex system,[12] intravalley oscillations[13] and intervalley transitions[12,14] of the vortex lines, thermally assisted flux flow.[15] However, Geshkenbein *et al.*[7] attributed these peculiarities namely to the size effect. Thus the size effect should be taken into account in the interpretations of all experiments at ac bilateral excitation.

Usually, the size effect is explained in the following way. Let us consider the bilateral, symmetric in ac magnetic field (antisymmetric in ac electric field), excitation of a superconducting plate. If the penetration depth δ of the ac field is much less than the plate thickness d, the impedance Z is small as $\delta/d \ll 1$ and increases with δ. In the opposite case when $\delta \gg d$, the ac electric field is small because it has different signs at the plate boundaries and does not vary practically within the whole sample volume. Therefore, the value of R turns out to be small again and decreases as δ is increased. This means that R should have a maximum at $\delta \sim d$. Thus in order for the size effect to take place, it is important to have two ac signals irradiating both sample boundaries. For this reason there exists an agreed-upon point of view that the size effect can be observed only at bilateral, symmetric in the external ac magnetic field, excitation.

The aim of this letter is to show that the size effect takes place at unilateral excitation just as at bilateral one. The external ac signal and the wave reflected from the interface superconductor substrate play the roles of two necessary waves to observe the size effect.

This fact is not only of fundamental interest, but holds significance for applications. Indeed, a lot of experiments are performed namely at the unilateral excitation and all ac experiments with superconducting films concerning this problem.

We shall demonstrate the size effect at unilateral excitation by the following simple example. Let a superconducting infinite plane-parallel plate overlies a substrate with dielectric constant ϵ. The x axis is normal to the plate boundaries. The free boundary of the superconductor is at $x=0$ and the interface superconductor substrate at $x=d$. The boundary $x=0$ is irradiated by an external ac magnetic field, directed along the plate surface (along the z axis) and having the form

$$H(t)=H_0 \cos(\omega t). \tag{1}$$

The magnetic induction $\vec{B}(x,t)$ contains the z component and the electric field $\vec{E}(x,t)$ has the y component only.

The electrodynamic properties of the superconductor will be described within the critical state model[16] which is valid in a wide range of ac amplitudes and frequencies (see, for example, Ref. 17). In the assumed geometry the Maxwell equations are written as

$$-\frac{\partial B}{\partial x}=\frac{4\pi\mu}{c}\, j_c \,\mathrm{sgn}\, E, \quad \frac{\partial E}{\partial x}=-\frac{1}{c}\frac{\partial B}{\partial t}. \tag{2}$$

Here, j_c is the critical current density and $\mu=1$ for monocrystals and films. For ceramic samples j_c is the critical density of intergranular current and factor μ represents the effective magnetic permeability $(0<\mu<1)$ which allows for intragranular currents.[18] The first equation in Eqs. (2) is valid only in those sample regions where the electric field is nonzero. In other regions, where $E=0$, the magnetic induction $B(x,t)$ turns out to be frozen. It keeps the same shape as at the last moment of prehistory when $E\neq 0$. The complete system for determining fields $E(x,t)$ and $B(x,t)$ is formed by Eqs. (2) together with boundary conditions

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$$B(0,t)=\mu H(t), \quad E(d,t)=B(d,t)/\mu\varepsilon^{1/2}\ . \tag{3}$$

The set of Eqs. (2) and (3) is essentially nonlinear. Therefore, the penetration depth δ of the ac field depends on the wave amplitude H_0, and we can study the manifestation of the size effect in the peculiarities of the surface impedance and absorptivity as functions of H_0. It should be noted that in the case of unilateral excitation the real part R of the surface impedance and the absorptivity A characterize different physical quantities. The value of R is connected to the wave energy passing through the surface $x=0$, whereas the value of A is related to the part of this energy dissipating within the superconducting plate $0<x<d$.

In this brief communication we present only the final results of calculations. At small ac amplitudes the size effect does not manifest itself because $\delta<d$ and the electromagnetic signal does not achieve the interface $x=d$. The corresponding inequality may be written as

$$H_0<H_p, \quad H_p=4\pi j_c d/c. \tag{4}$$

Here, H_p represents the ac amplitude at which the radiowave reaches the substrate ($\delta=d$). The formulas for the surface impedance and absorptivity at $H_0<H_p$ can be expressed via the ratio $h_0=H_0/H_p$:

$$R(h_0)/R(1)=X(h_0)/X(1)=A(h_0)/A(1)=h_0,$$
$$R(1)=4X(1)/3\pi=\pi A(1)/c=8\mu\omega d/3c^2. \tag{5}$$

At high amplitudes,

$$H_0>H_p, \quad h_0>1, \tag{6}$$

the size effect occurs ($\delta>d$) and the functions $Z(h_0)$ and $A(h_0)$ become sensitive to the dielectric properties of the substrate, i.e., to the value of ϵ. Note that ϵ can vary in a broad range because even ferroelectrics are used as substates.[19] The shape of the functions $Z(h_0)$, $A(h_0)$ at $h_0>1$ is governed by the parameter

$$\alpha=\mu\omega\varepsilon^{1/2}d/c, \tag{7}$$

which represents the absolute value of the ratio of the surface impedances of the superconductor plate (at $h_0=1$) and substrate.

The asymptotics for $Z(h_0)$, $A(h_0)$ at $\alpha\gg 1$, $h_0>1$ are

$$\frac{R(h_0)}{R(1)}=\frac{A(h_0)}{A(1)}=\frac{3}{h_0}-\frac{2}{h_0^2}; \tag{8}$$

$$\frac{X(h_0)}{X(1)}=2-\frac{2-h_0}{\pi}\arccos\left(1-\frac{2}{h_0}\right)+\frac{4}{\pi}\left(1-\frac{2}{h_0}\right)^2 \times(h_0-1)^{1/2}-\frac{16}{3\pi h_0^2}(h_0-1)^{3/2}. \tag{9}$$

Note that these asymptotics can be realized, for example, if a metal serves as the substrate. In this situation, the size effect results in maxima of $R(h_0)$ and $A(h_0)$ at $h_0=4/3$.

In the opposite case when $\alpha\ll 1$, $h_0>1$ the following formulae are valid,

$$\frac{R(h_0)}{R(1)}=\frac{3}{\alpha}\arccos(h_0^{-1})-\frac{3(h_0^2-1)^{1/2}}{a h_0^2}+\frac{1}{h_0^2}; \tag{10}$$

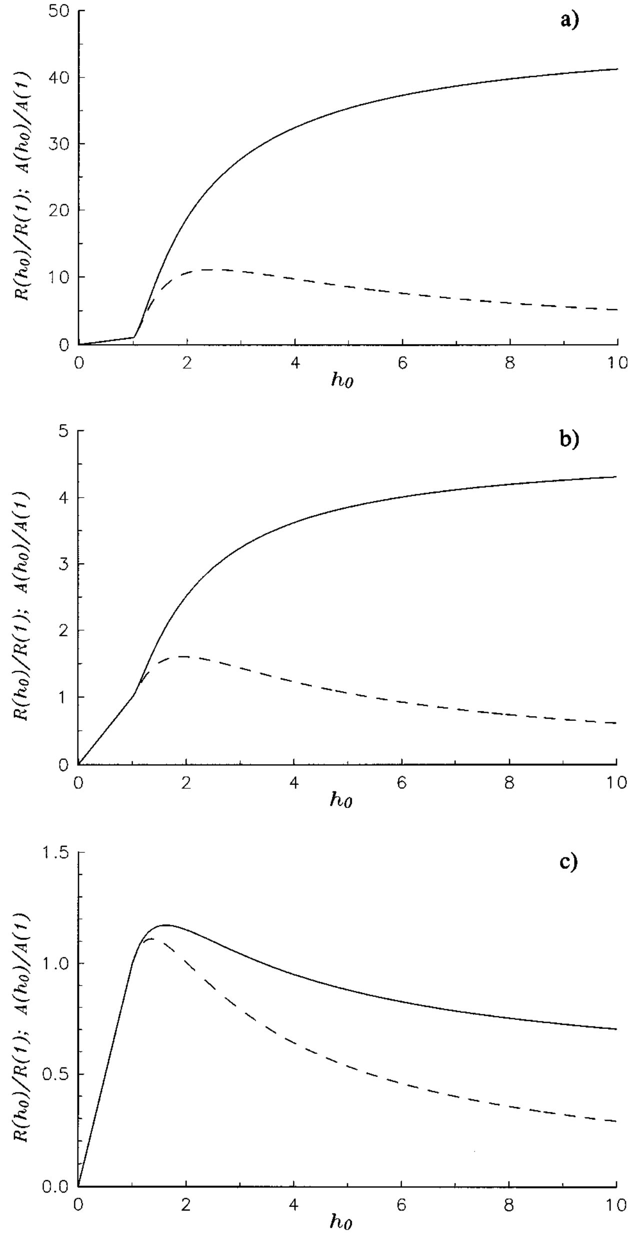

FIG. 1. Dependences $R(h_0)/R(1)$ (solid line) and $A(h_0)/A(1)$ (dashed line) at (a) $\alpha=0.1$, (b) $\alpha=1.0$, (c) $\alpha=10$.

$$\frac{X(h_0)}{X(1)}=1+\frac{2}{\pi}\arccos(h_0^{-1})-\frac{2}{\pi}\frac{(h_0^2-1)^{1/2}}{h_0^2}; \tag{11}$$

$$\frac{A(h_0)}{A(1)}=\frac{6}{\alpha h_0^2}[(h_0^2-1)^{1/2}-\arccos(h_0^{-1})]+\frac{1}{h_0^2} \tag{12}$$

The size effect leads to a high maximum with a magnitude of the order of α^{-1} in $A(h_0)$ dependence, an abrupt increase of $R(h_0)$ and a break of $X(h_0)$ at $h_0\sim 1$. Numerically calculated plots for $R(h_0)$, $X(h_0)$, and $A(h_0)$ are presented in Figs. 1 and 2. It is seen that the maximum in $A(h_0)$ dependence exists at any value of the parameter α. Note, that the argument h_0 is defined not only by the ac amplitude H_0 but also by j_c. So, the revealed peculiarities should be observed in temperature dependences of Z and A as well.

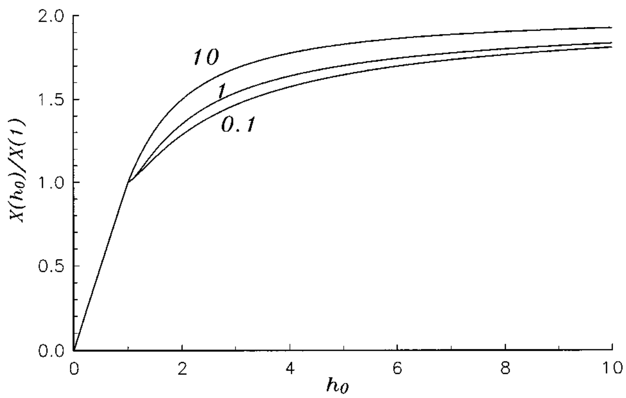

FIG. 2. Dependences $X(h_0)/X(1)$ at different values of α indicated near curves.

Thus, we have shown the size effect at the unilateral ac excitation within the critical state model. Of course, similar results may be obtained in the framework of any other model. It should be note that many authors study the fundamental properties of hard superconductors using ac measurements at unilateral excitation. For instance, Gasparov[20] investigated Berezinskii–Kosterlitz–Thouless transition in YBaCuO single crystal films by this method (among others). The results of our work demonstrate that the size effect should be taken into account in such a kind of experiments.

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