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JUMPS OF THE ELECTRIC FIELD ON THE SURFACE OF A HARD SUPERCONDUCTOR

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The nonlinear interaction of two low-frequency radio waves in hard superconductors is studied theoretically and experimentally. The jumps of the temporal dependence of the electric field at a sample surface caused by this interaction are predicted and detected. The necessary conditions to observe this phenomenon are formulated.

Keywords: A. high-T_c superconductors, D. anharmonicity.

1. INTRODUCTION

NONLINEARITY of the material equation specifying the relationship between the current density and the electromagnetic field in conducting media results in a violation of the superposition principle and leads to the interaction of electromagnetic waves inside a sample bulk. The specific character of the nonlinearity is displayed in the features of such an interaction. For example, the so called magnetodynamic mechanism of nonlinearity, connected with the influence of a wave on the dynamics of charge carriers, plays an extremely important role in electromagnetic processes in pure metals at low temperatures [1]. It results in a specific interaction between two radio waves which does not have analogues in other nonlinear media. As a result, a rather unusual hysteretic dependence of the low frequency surface impedance on the amplitude of a high frequency wave is observed in pure metals [2, 3].

A study of features of the nonlinear electromagnetic wave interaction in hard superconductors is of great interest. The electrodynamic properties of such materials are described by the critical state model [4, 5] in a rather wide interval of wave amplitudes and frequencies (see, for example, [6]). In the frame of this model, the Maxwell equation specifying the distribution of a magnetic induction **B** inside a superconductor has the following form,

$$\operatorname{rot}\mathbf{B} = \frac{4\pi}{c} j_c(B)\frac{\mathbf{E}}{E}. \tag{1}$$

Here **E** is the electric field, $j_c(B)$ is the critical current density, c is the speed of light. The critical state equation is essentially nonlinear. The nonlinearity in the right-hand side of equation (1) is stipulated by the dependence $j_c(B)$, as well as by the multiplier $\mathbf{E}/E$. The nonlinearity connected with this multiplier is of principal interest, since it does not occur in any other nonlinear media. Owing to this nonlinearity unusual effects are observed in hard superconductors (see, for example, [7, 8]). It is shown below that this nonlinearity results in a peculiar phenomenon in the radio wave interaction, namely, in jumps of the electric field at the sample surface vs time. This effect is very pronounced in high-T_c ceramics which represent specific nonlinear Josephson media. As it was shown in [9], the electromagnetic properties of such superconductors may be described by an equation similar to equation (1).

2. THEORY

Let us consider a superconducting plate of thickness d placed into an external a.c. magnetic field having the following form

$$H(t) = H_1 \cos(\omega_1 t) + H_2 \cos(\omega_2 t + \alpha). \tag{2}$$

This field is directed in parallel to a plate surface (along the axis z). The magnetic induction $\mathbf{B}$ and the electric field $\mathbf{E}$ depend on a single spatial coordinate x in this geometry. The axis x is directed along the plate normal and the plane $x = 0$ is located in the middle of the sample. The vector $\mathbf{B}$ contains only a z-component and the vector $\mathbf{E}$ only a y-component. In this situation the function $B(x)$ is even, whereas $E(x)$ is odd. The electric field $E(t) \equiv E(d/2, t)$ at the surface of the superconductor is related to the derivative of the magnetic flux Φ per unit plate width with respect to time,

$$E(t) = -\frac{1}{2c}\frac{\mathrm{d}\Phi}{\mathrm{d}t}, \qquad \Phi = \int_{-d/2}^{d/2} B(x)\,\mathrm{d}x. \tag{3}$$

The considerations below are based on the critical state model. In our geometry the critical state equation (1) can be rewritten as follows

$$\frac{\mathrm{d}B}{\mathrm{d}x} = \frac{4\pi}{c} j_c(B)\,\mathrm{sign}\,E. \tag{4}$$

It should be noted that this equation is valid only in those sample regions where the electric field differs from zero. In other regions the distribution of the magnetic induction $B(x)$ proves to be frozen. It keeps the same shape as at the last moment of prehistory when $E \neq 0$.

Let us follow the evolution of the distribution $B(x)$ inside the plate placed into the external field (2). For the sake of simplicity we shall consider the frequency of the first wave to be significantly less than that of the second wave, $\omega_1 \ll \omega_2$. At some moment $t = t_0$ the magnetic field at the sample surface reaches its maximum value $H_{\mathrm{max}}^{(0)}$. According to the critical state model, at that point of time the electric field is equal to zero everywhere inside the superconductor, and the magnetic induction is distributed in a way shown schematically in Fig. 1(a) by a thick solid line. With time, the magnetic field $H(t)$ at the sample surface decreases and the electric field rises inside the superconductor. As a result, the plate becomes divided into three regions. The electric field differs from zero on the intervals $\bar{x} < |x| < d/2$. Here the sign of the derivative $\mathrm{d}B/\mathrm{d}x$ is opposite to the one at the initial moment $t = t_0$. In the region $|x| < \bar{x}$ the electric field remains zero. The distribution $B(x)$ holds the form which it had at the moment $t = t_0$ here. A sequence of plots demonstrating the change of the distribution $B(x)$ with time is shown in Fig. 1 by broken lines 1–3. At some moment $t = t_1$ the field $H(t)$ at the sample surface reaches its minimum value $H_{\mathrm{min}}^{(1)}$. The electric field becomes zero everywhere inside the sample at that point of time, and the distribution $B(x)$ takes the form shown by curve 3 in Fig. 1(a). The distribution $B(x)$ varies with time in the interval $t_1 \leq t \leq t_2$, as it is shown in Fig. 1(b). At a moment $t = t_2$ the field $H(t)$ at the sample surface reaches its new maximum value $H_{\mathrm{max}}^{(2)}$ which is less than the initial maximum value $H_{\mathrm{max}}^{(0)}$. The following half-cycle $t_2 \leq t \leq t_3$ of the high-frequency wave is very substantial for our analysis. During this time the form of the distribution $B(x)$ changes consistently from the plot 1 up to 4 in Fig. 1(c). It is important that the new minimum value $H_{\mathrm{min}}^{(3)}$ of $H(t)$ at $t = t_3$ is less than $H_{\mathrm{min}}^{(1)}$. Figure 1(c) shows that a jump of the derivative of the magnetic flux with respect to time occurs when the field $H(t)$ passes through the value $H_{\mathrm{min}}^{(1)}$. Curve 3 in Fig. 1(c) corresponds to this moment. As a matter of fact, at that point the positions of the planes $|x| = \bar{x}$, dividing the regions where $\partial B/\partial t = 0$ and $\partial B/\partial t \neq 0$, changes abruptly. It means that in a given situation a jump of the electric field $E(t)$ at the sample surface should be observed [see equation (3)].

Similar change of the spatial distribution of the magnetic induction takes place in cylindrical samples also. Hence, jumps in the electric field $E(t)$ should be observed in this geometry as well.

These jumps exist not only in the case $\omega_1 \ll \omega_2$. However, their occurrence depends on amplitudes and frequencies of the interacting electromagnetic waves, as well as on their initial phase difference α. We can formulate two necessary conditions for the existence of $E(t)$ jumps. These conditions are connected with some requirements to the external magnetic field $H(t)$ in equation (2). It must have several maxima (or minima) with different heights. Besides, the function $H(t)$ should have pairs of neighboring maxima and minima, whose heights H_{max} and H_{min} satisfy the condition

$$H_{\mathrm{max}} - H_{\mathrm{min}} < 2H_p. \tag{5}$$

Here

$$H_p = \frac{2\pi}{c} j_c d, \tag{6}$$

represents the value of the external magnetic field at which the magnetic flux penetrates up to the middle of the sample. If equation (5) is invalid for all neighboring maxima and minima of $H(t)$, the necessary initial profile of $B(x)$, similar to curve 1 in Fig. 1(c), will never be formed owing to the full penetration of the a.c. field into a sample.

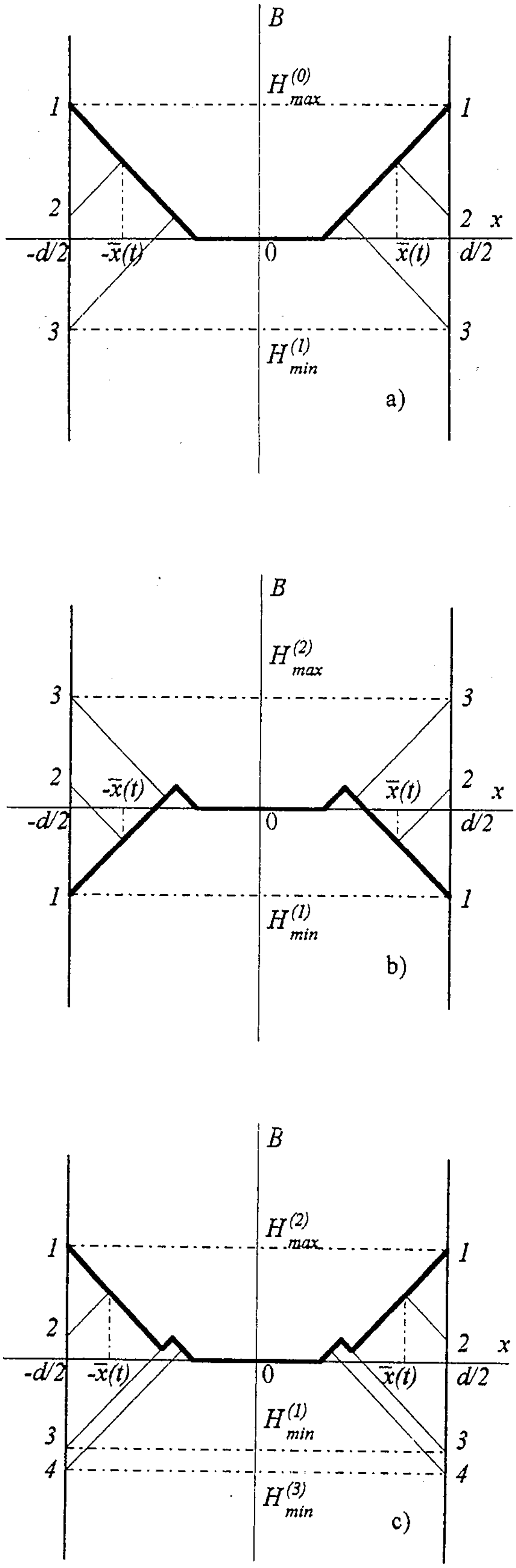

Fig. 1. Evolution of the spatial distribution of the magnetic induction $B(x)$ in different time intervals: (a) $t_0 \le t \le t_1$, (b) $t_1 \le t \le t_2$, and (c) $t_2 \le t \le t_3$.

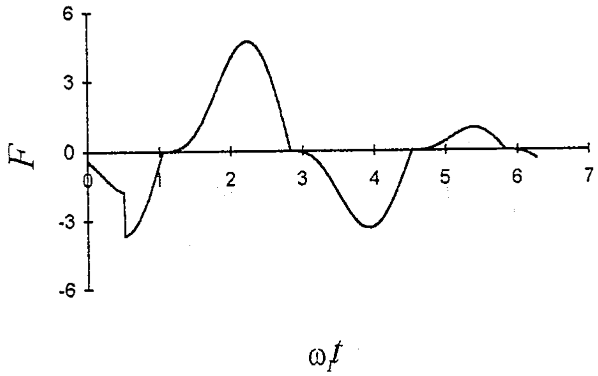

Fig. 2. The calculated dependence of the dimensionless electric field F on $\omega_1 t$.

The mathematical treatment gives the following result for $E(t)$ at the surface of a superconducting plate,

$$E(t) = \frac{1}{4\pi j_c}\left|\frac{\mathrm{d}H(t)}{\mathrm{d}t}\right|[\bar{B}(t) - H(t)]. \tag{7}$$

Here $\bar{B}(t)$ represents the magnetic induction B at the points $|x| = \bar{x}(t)$. Accordingly, the height of the $E(t)$ jump can be expressed through the jump $\Delta\bar{B}$ of the field $\bar{B}(t)$

$$\Delta E = \frac{1}{4\pi j_c}\left|\frac{\mathrm{d}H(t)}{\mathrm{d}T}\right|\Delta\bar{B}. \tag{8}$$

The results of equations (7) and (8) as well as the schematic representation of the distribution $B(x)$ are presented for the case $j_c(B) = \text{const}$. However, the origin of the described phenomenon is not connected to this condition, i.e. the jumps of $E(t)$ should be observed also in hard superconductors with $j_c \neq \text{const}$. It is clear that taking into account the particular magnetic-field dependence of the critical current density is required for the quantitative calculation of the field $E(t)$.

To illustrate the discussed phenomenon we present the result of the numerical calculation of dimensionless electric field $F(\omega_1 t)$ performed on the basis of equation (4) with $j_c = \text{const}$ for a cylindrical sample at $H_1/H_p = 0.19$, $H_2/H_p = 0.27$, $\omega_2/\omega_1 = 2$, $\alpha = 3.9$ in Fig. 2. The quantity $F(\omega_1 t)$ is connected with the electric field $E(t)$ by the following relation

$$E(t) = \frac{\omega_1 H_1^2}{4\pi j_c} F(\omega_1 t). \tag{9}$$

3. EXPERIMENT

High-T_c ceramics are the most suitable object for the observation of nonlinear phenomena connected with the nonlinearity of the critical state equation. Therefore, for the experimental study of the wave

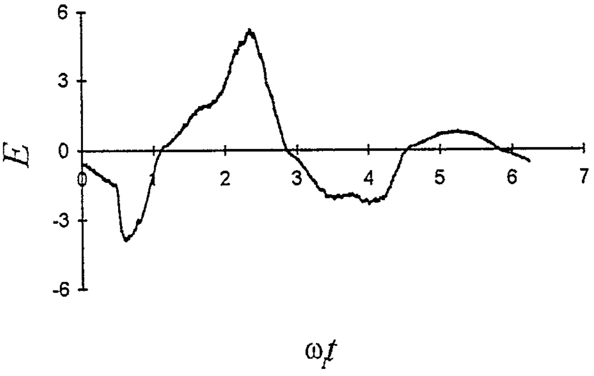

Fig. 3. The measured dependence of E vs $\omega_1 t$.

interaction samples of yttrium ceramics prepared by means of the standard technology from initial oxides Y_2O_3, $BaCO_3$ and CuO have been chosen. The cylindrical samples 5 mm in diameter had a length about 3 cm. The typical value of the critical current density was $\sim 100\,A\,cm^{-2}$. The penetration field H_p was about 28 Oe. The a.c. magnetic field (2) was created by a solenoid made with a copper wire. To measure a signal proportional to $E(t)$ a monolayer pick-up coil wound directly on the sample (copper wire 30 μm in diameter) was used. The signal was preamplified by a PAR-124A amplifier working in a broadband mode. The measurement process was computer controlled, allowing rapid data acquisition and calculations.

The abrupt jumps of the electric field discussed above were observed for a wide range of parameters of the a.c. field (2). A pattern of the experimental dependence $E(t)$ obtained for $\omega_1/2\pi = 160$ Hz, $\omega_2/2\pi = 320$ Hz, $H_1 = 5.3$ Oe, $H_2 = 7.54$ Oe, $\alpha = 3.9$ is presented in Fig. 3 in arbitrary units. This figure shows clearly the abrupt jump of the electric field at $\omega_1 t = 0.5$. The direct comparison of the theoretical curve in Fig. 2, calculated at the same conditions as the experimental results shows good qualitative agreement. Some deviations may be connected with the magnetic field dependence of the critical current density which we did not take into account in our calculations.

In this brief paper we have restricted ourselves to the discussion of the origin of the jumps in $E(t)$ for radio wave interaction, as well as their experimental detection. To analyze this phenomenon in detail we need to construct a theory of the wave interaction in hard superconductors using the critical state model with an arbitrary dependence of the critical current density on the magnetic field, as well as to carry out some additional experimental research.

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