Effect of the substrate on the ac response of superconductors with strong pinning to an incident plane wave

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The influence of the electromagnetic properties of the substrate upon the size effect in hard superconductors is theoretically studied. Within the frame of the critical state model we calculate the distribution of the magnetic induction and the electric field inside a superconducting plate subject to unilateral electromagnetic excitation. This calculation allows to obtain the dependences of surface impedance and absorptivity on the amplitude of the external signal. We find that the manifestation of the size effect in the ac response of the superconducting plate depends strongly on the value of the dielectric constant of the substrate.
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I. INTRODUCTION

In view of the discovery of the high-temperature superconductivity the investigations on electromagnetic properties of hard superconductors have been intensified. So, a great variety of electrodynamic phenomena is actually studied. Among these phenomena, the size effect in superconductors is of particular interest. On its basis, for example, the new contactless method[1,2] for measuring the critical parameters of superconducting materials was developed.

Frequently, the size effect is analyzed in the case of the bilateral excitation of a superconducting plate.[2–7] Its origin is explained in the same way as is done for normal metals.[8,9] When penetration depth δ of the ac field in the sample is much larger than the plate thickness d ($\delta \gg d$), the electric field is notably diminished since it is a smooth function of coordinates and has opposite signs at the plate boundaries. As a consequence, the absorptivity decreases with increase of δ. On the other hand, when δ is much less than d the surface resistance is small as $\delta/d \ll 1$ and increases as δ is augmented. This means that the real part of surface impedance has a maximum in the region $\delta \sim d$ (size effect).

Recently it was shown that the size effect takes place at unilateral excitation as well.[10] As at bilateral excitation, there are two waves in the sample that give rise to the size effect: the external ac signal and the wave reflected from the interface superconductor-substrate. This fact is important because many experiments (see, for example, Ref. 11) are carried out precisely at the unilateral excitation and it appertains to all ac experiments with superconducting films. In this kind of experiments usually the dependence of the surface impedance Z on external parameters (temperature, magnetic field, frequency, amplitudes), which determine the value of the penetration depth δ, is measured. So, maxima in the absorptivity (ReZ) can be interpreted as a manifestation of the size effect. Nevertheless, a lot of experiments with plates (or with films) are carried out for the system plate-substrate. Evidently the electromagnetic properties of the substrate affect notably the distribution of the field inside the superconducting plate and, consequently, determine its absorptivity (surface resistance). Therefore, the size effect should depend strongly on the kind of the substrate (semiconductor, metal, perovskite-structure oxide, etc.[12]). It is noteworthy that the measurements of the surface resistance of high-temperature superconductors can be performed both in linear (see, for instance, Refs. 13–17) and in nonlinear regimes.[1,2,18,19] Thus, in both cases the size effect should be considered when the penetration depth of the ac field is of the same order as the plate thickness.

The aim of the present work is to investigate the influence of the substrate on the size effect in hard superconductors at unilateral excitation. This work provides the full theoretical framework to our letter.[10] So, we shall study the ac response of a superconducting plate by using the critical state model which can be applied in a wide range of ac amplitudes and frequencies (see, for example, Ref. 20), characterizing the nonlinear region. In Sec. II problem statement and fundamental equations are presented. In Secs. III and IV we obtain formulae for the distribution of the electromagnetic field inside the superconductor in the cases of small and large amplitudes of the radio wave, respectively. Finally (Sec. V), we calculate numerically the electric field at the plate surfaces, the surface impedance and absorptivity. We compare these results with analytical ones of previous sections.

II. PROBLEM STATEMENT

Let us consider a superconducting infinite plane-parallel plate of thickness d which overlies a substrate with dielectric constant ε. The x-axis is normal to the plate boundaries. The free boundary of the superconductor is at $x=0$ and the interface superconductor–substrate at $x=d$. The free boundary

$x=0$ is irradiated by an external ac magnetic field, directed along the plate surface (along the z-axis) and having the form

$$H(t)=H_0 \cos(\omega t). \tag{2.1}$$

The magnetic induction $\mathbf{B}(x,t)$ and the electric field $\mathbf{E}(x,t)$ depend on a single spatial coordinate x in this geometry. Vectors $\mathbf{B}$ and $\mathbf{E}$ are parallel to z- and y-axes, respectively:

$$\mathbf{B}(x,t)=\{0,0,B(x,t)\}; \quad \mathbf{E}(x,t)=\{0,E(x,t),0\}. \tag{2.2}$$

Considering that a shift $\Delta t=\pi/\omega$ in the time dependences of the fields $B(x,t)$ and $E(x,t)$ changes solely their signs, it is enough to analyze these functions only in the first half-period,

$$0\leqslant\omega t\leqslant\pi. \tag{2.3}$$

We shall calculate the electric field $E(x,t)$ at the boundary $x=0$ and at the interface $x=d$, thereafter the surface impedance,[1,2]

$$Z=\frac{8\omega}{cH_0}\int_0^{\pi/\omega} dt E(0,t)\exp(\mathrm{i}\omega t). \tag{2.4}$$

Here c is the speed of light in vacuum. Also, we will obtain the plate absorptivity A which is defined as the ratio of the difference of the irradiances at the surface $x=0$ and the interface $x=d$ to the incident irradiance:

$$A=\frac{\langle E(0,t)H(0,t)\rangle-\langle E(d,t)H(d,t)\rangle}{H_0^2/8}, \tag{2.5}$$

where brackets $\langle \dots \rangle$ denote time averaging, $H(0,t)=H(t)$ (2.1). Eq. (2.5) can be rewritten in the form

$$A=\frac{8\omega}{\pi H_0}\int_0^{\pi/\omega} dt[E(0,t)\cos(\omega t)-\varepsilon^{1/2}E^2(d,t)/H_0]. \tag{2.6}$$

It should be noted that in the case of unilateral excitation the real part R of the surface impedance and the absorptivity A characterize different physical quantities. The value of R is connected to the wave energy passing through the surface $x=0$, whereas the value of A is related to the part of this energy dissipating within the superconducting plate $0<x<d$.

According to the assumed geometry, the Maxwell equations within the critical state model[21,22] can be written as follows:

$$\frac{\partial E}{\partial x}=-\frac{1}{c}\frac{\partial B}{\partial t}; \quad -\frac{\partial B}{\partial x}=\frac{4\pi\mu}{c}j_c \text{ sign } E. \tag{2.7}$$

In the last equation j_c is the critical current density and $\mu=1$ for monocrystals and films. For ceramic samples j_c is the critical density of intergranular current and factor μ represents the effective magnetic permeability $(0<\mu<1)$ which allows for intragranular currents preventing magnetic flux penetration into the grains. Here, we shall not take into account the dependence of j_c on B ($j_c=$ const). It should be noted that the critical state equation in (2.7) is valid if the London penetration depth λ is the smallest parameter with dimensions of length, and if the inequality $H_0>4\pi j_c\lambda/c$ (nonlinear region) is satisfied. The critical state equation is applied only in those sample regions where the electric field is nonzero. In other regions, where $E=0$, the distribution of the magnetic induction $B(x,t)$ turns out to be frozen. It keeps the same shape as at the last moment of prehistory when $E\neq 0$.

Boundary conditions for Eqs. (2.7) are

$$B(0,t)=\mu H(t), \quad E(d,t)=B(d,t)/\mu\varepsilon^{1/2}. \tag{2.8}$$

Let us introduce dimensionless variables

$$h_0=H_0/H_p, \quad \xi=x/d, \quad 0\leqslant\xi\leqslant 1, \quad \tau=\omega t; \tag{2.9}$$

and dimensionless functions

$$b(\xi,\tau)=B(x,t)/\mu H_p, \quad F(\xi,\tau)=(c/\mu\omega d H_p)E(x,t). \tag{2.10}$$

Here $H_p=4\pi j_c d/c$ represents the characteristic value of the wave amplitude H_0 at which the ac magnetic field penetrates the whole volume of the superconductor up to the interface $x=d$ $(\xi=1)$.

The new notations allow to rewrite equations (2.7) and boundary conditions (2.8) as

$$\partial F/\partial\xi=-\partial b/\partial\tau, \quad \partial b/\partial\xi=-\text{sign } F;$$

$$b(0,\tau)=h_0 \cos \tau, \quad b(1,\tau)=\alpha F(1,\tau), \quad \alpha=\mu\omega\varepsilon^{1/2}d/c. \tag{2.11}$$

As will be seen in Sec. IV the parameter α characterizes the role of the substrate in the ac response of the superconducting plate. One can see that the critical state equation in Eq. (2.7) and Eq. (2.11) does not contain the displacement current. A simple comparison of this current with j_c shows its smallness in accordance with the parameter $4\pi\omega d h_0/c\ll 1$. Since the value of h_0 in our consideration is about unity, neglecting of the displacement current is provided by the inequality $\omega d/c\ll 1$, usual for the quasistationary situation.

The solution of the formulated problem is quite distinct depending on whether the magnetic flux penetrates the whole plate volume or not. For this reason, we shall consider separately the cases of small $(H_0<H_p)$ and large $(H_p<H_0)$ amplitudes of the external electromagnetic wave.

III. SMALL AMPLITUDES OF THE RADIO-WAVE

If the amplitude of ac magnetic field is less than the penetration field,

$$H_0\leqslant H_p, \quad \text{i.e.,} \quad h_0\leqslant 1, \tag{3.1}$$

the magnetic flux does not pass through the whole superconductor volume, and induction $B(x,t)$ at the interface $x=d$ is zero. Evidently, in this case there is no size effect and the substrate properties do not manifest themselves in the behavior of fields $B(x,t)$ and $E(x,t)$.

The magnetic induction $b(\xi,\tau)$ and the electric field $F(\xi,\tau)$ within the superconducting plate are easily obtained by solving equations (2.11). We get

$$b(\xi,\tau)=\begin{cases}\xi+h_0\cos\tau, & 0\leqslant\xi\leqslant\tilde{\xi}(\tau);\\ -\xi+h_0, & \tilde{\xi}(\tau)\leqslant\xi\leqslant h_0;\\ 0, & h_0\leqslant\xi\leqslant 1,\end{cases}$$

$$F(\xi,\tau)=\begin{cases}-h_0\sin\tau(\tilde{\xi}(\tau)-\xi), & 0\leqslant\xi\leqslant\tilde{\xi}(\tau);\\ 0, & \tilde{\xi}(\tau)\leqslant\xi\leqslant 1,\end{cases}\qquad(3.2)$$

$$\tilde{\xi}(\tau)=h_0\sin^2(\tau/2).$$

Since the electromagnetic energy is not transmitted to the substrate ($F(1,\tau)=0$), the absorptivity (2.6) of the superconductor turns out to be proportional to the real part of the surface impedance (2.4),

$$A=(c/\pi)\mathrm{Re}Z,\qquad(3.3)$$

which is straightforwardly calculated by using formulas (3.2):

$$Z=\frac{8\mu\omega h_0 d}{3c^2}\left(1-\frac{3\pi\mathrm{i}}{4}\right).\qquad(3.4)$$

As is seen, in the case of small amplitudes (3.1) the results for unilateral and bilateral excitations of a superconducting plate agree (see, for example, Refs. 1 and 2). So, in this nonlinear regime the surface impedance turns out to be proportional to the amplitude h_0.

IV. LARGE AMPLITUDES OF THE RADIO-WAVE

The study of the region of large amplitudes,

$$H_p\leqslant H_0,\quad\text{i.e.,}\quad 1\leqslant h_0,\qquad(4.1)$$

is of great interest since precisely here the size effect takes place, being notably affected by the electromagnetic properties of the substrate. In the case (4.1) the electromagnetic signal goes through the superconductor and decays inside the substrate medium. Magnetic induction $B(x,t)$ together with electric field $E(x,t)$, in general, are different from zero in the whole bulk of the superconducting plate and at the interface $x=d$.

According to set (2.11), at the initial (starting) time moment $\tau=0$ dimensionless induction $b(\xi,\tau)$ has a maximum ($\partial b(\xi,\tau)/\partial\tau=0$ at $\tau=0$). Its spatial behavior inside the plate is illustrated by straight line 1 in Fig. 1. With increasing ξ, function $b(\xi,0)$ decreases from $b(0,0)=h_0$ at the free boundary $\xi=0$ up to the positive value $b(1,0)=h_0-1$ at the interface $\xi=1$. Beginning from this moment and during the whole half-period (2.3), the induction decreases inside the superconductor ($\partial b(\xi,\tau)/\partial\tau<0$ at $0<\tau<\pi$). Nevertheless, that decrease, as well as the spatial distribution of $b(\xi,\tau)$, will have qualitatively distinct behaviors in different time intervals. In order to clarify this, let us integrate the first equation from set (2.11) with respect to ξ:

$$F(\xi,\tau)=F(1,\tau)+\int_\xi^1 d\xi'\,\frac{\partial b(\xi',\tau)}{\partial\tau}.\qquad(4.2)$$

In accordance with the boundary condition at $\xi=1$ from set (2.11), at $\tau=0$ the electric field at the interface $\xi=1$ is positive ($F(1,0)=(h_0-1)/\alpha>0$). Besides, at this time-moment the second term in the right-hand side of Eq. (4.2) is

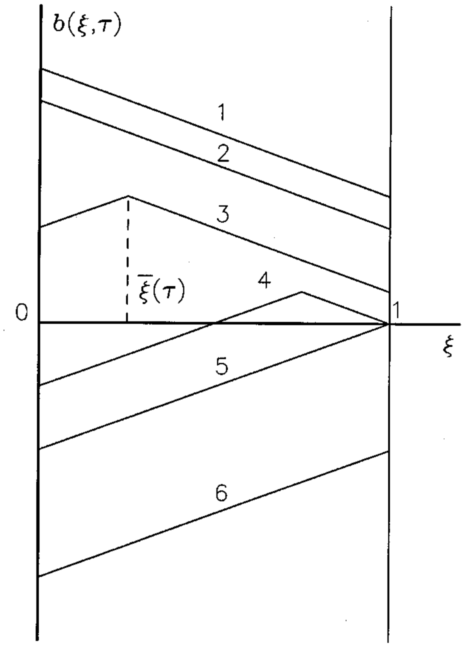

FIG. 1. The spatial distribution of the dimensionless magnetic induction $b(\xi,\tau)$ inside the plate for different time moments: (1) $\tau=0$, (2) $\tau=\tau_1$, (3) and (4) $\tau_1<\tau<\tau_2$, (5) $\tau=\tau_2$, (6) $\tau=\pi$.

equal to zero everywhere in the superconductor and, consequently, $F(\xi,0)=F(1,0)>0$. Subsequently, in the whole half-period (2.3) the second term in Eq. (4.2) is negative and the field at the interface $F(1,\tau)$ decreases with τ. Under these conditions the electric field $F(\xi,\tau)$ will decrease also. It is clear that it will conserve its positive sign ($\mathrm{sign}\,F=1$) only up to certain time moment τ_1, when the field at the free boundary $\xi=0$ vanishes ($F(0,\tau_1)=0$). After solving set (2.11) in the interval $0\leqslant\tau\leqslant\tau_1$, we get:

$$b(\xi,\tau)=-\xi+h_0\cos\tau,$$

$$F(\xi,\tau)=-h_0(1-\xi)\sin\tau+(h_0\cos\tau-1)/\alpha,$$

$$0\leqslant\tau\leqslant\tau_1.\qquad(4.3)$$

Using these expressions one can rewrite the equation for τ_1, i.e., $F(0,\tau_1)=0$, in the form

$$\cos\tau_1-\alpha\sin\tau_1=h_0^{-1}.\qquad(4.4)$$

Hence, in the first time interval $0\leqslant\tau\leqslant\tau_1$ the spatial distribution of induction $b(\xi,\tau)$ has a linear behavior (4.3). With time the plot of $b(\xi,\tau)$ moves parallel from the initial straight line $b=b(\xi,0)$ to the side of smaller values of b, and, finally, is transformed into the straight line $b=b(\xi,\tau_1)$. This line is labeled by number 2 in Fig. 1. Consequently, in the first stage when $0\leqslant\tau\leqslant\tau_1$ the electric field in the superconductor decreases remaining positive everywhere.

The second stage begins at $\tau=\tau_1$ when the electric field $F(0,\tau_1)$ at the irradiated surface $\xi=0$ of the plate is equal to zero, being positive in the superconductor and at the interface $\xi=1$. With time, the spatial distribution of $F(\xi,\tau)$ becomes sign variable: in the region $0\leqslant\xi\leqslant\tilde{\xi}(\tau)$ near surface

$\xi=0$ it is negative (sign $F=-1$), and close to the interface at $\overline{\xi}(\tau)\leqslant\xi\leqslant 1$, remains positive (sign $F=1$). According to the equation of the critical state from set (2.11), the spatial distribution of magnetic induction $b=b(\xi,\tau)$ has the form of a step-linear function (see broken lines 3, 4 in Fig. 1). This stage finishes at $\tau=\tau_2$ when the electric field, and together with it the induction at the interface vanishes $(F(1,\tau_2)=b(1,\tau_2)/\alpha=0)$. Thus, everywhere in the superconductor (including its free boundary $\xi=0$) both fields $F(\xi,\tau_2)$ and $b(\xi,\tau_2)$ become negative, and the broken line degenerates into straight line 5 in Fig. 1 for $b=b(\xi,\tau)$. From set (2.11) formulae for the dimensionless induction $b=b(\xi,\tau)$ and the electric field $F(\xi,\tau)$ at the second interval of the time evolution take the form:

$$b(\xi,\tau)=\begin{cases}\xi+h_0\cos\tau, & 0\leqslant\xi\leqslant\overline{\xi}(\tau);\\ -\xi+1+b(1,\tau), & \overline{\xi}(\tau)\leqslant\xi\leqslant 1,\end{cases}$$

$$F(\xi,\tau)=\begin{cases}\dfrac{b(1,\tau)}{\alpha}-h_0\sin\tau(\overline{\xi}(\tau)-\xi)+\dfrac{\partial b(1,\tau)}{\partial\tau}(1-\overline{\xi}(\tau)), \\ \qquad 0\leqslant\xi\leqslant\overline{\xi}(\tau); \\ \dfrac{b(1,\tau)}{\alpha}+\dfrac{\partial b(1,\tau)}{\partial\tau}(1-\xi), \\ \qquad \overline{\xi}(\tau)\leqslant\xi\leqslant 1,\end{cases} \tag{4.5}$$

$$\tau_1\leqslant\tau\leqslant\tau_2.$$

From the continuity of magnetic induction $b(\xi,\tau)$ at point $\xi=\overline{\xi}(\tau)$ it is easy to get the relation between quantities $b(1,\tau)$ and $\overline{\xi}(\tau)$:

$$2\overline{\xi}(\tau)=b(1,\tau)+1-h_0\cos\tau. \tag{4.6}$$

There is an equation for the induction $b(1,\tau)$ at the interface, which follows from condition $F(\overline{\xi}(\tau),\tau)=0$. Assuming $\xi=\overline{\xi}(\tau)$ for $F(\xi,\tau)$ in Eq. (4.5) and using Eq. (4.6), we find

$$2b(1,\tau)+\alpha[1+h_0\cos\tau-b(1,\tau)]\partial b(1,\tau)/\partial\tau=0. \tag{4.7}$$

It is necessary to complement this equation with the initial condition at $\tau=\tau_1$, where $\overline{\xi}(\tau_1)=0$. From Eqs. (4.3), (4.4) or (4.6) one has:

$$b(1,\tau_1)=h_0\cos\tau_1-1=\alpha h_0\sin\tau_1. \tag{4.8}$$

The expression for electric field $F(\xi,\tau)$ (4.5) can be simplified by using equation (4.7):

$$F(\xi,\tau)=\begin{cases}-h_0(\overline{\xi}(\tau)-\xi)\sin\tau, & 0\leqslant\xi\leqslant\overline{\xi}(\tau);\\ \dfrac{b(1,\tau)}{\alpha}\,\dfrac{\xi-\overline{\xi}(\tau)}{1-\overline{\xi}(\tau)}, & \overline{\xi}(\tau)\leqslant\xi\leqslant 1.\end{cases} \tag{4.9}$$

Finally, the equation for the last time-moment τ_2 of the second stage is obtained from Eq. (4.6) by making there the substitutions $\overline{\xi}(\tau_2)=1$ and $b(1,\tau_2)=0$:

$$\cos\tau_2=-h_0^{-1}. \tag{4.10}$$

Third (final) stage in the half-period is similar to the first one. It occurs in the interval $\tau_2\leqslant\tau\leqslant\pi$ and is different from the first stage only in the sign of the electric field within the whole superconductor (sign$F=-1$). The initial straight line, $b=b(\xi,\tau_2)$, for this interval moves parallel towards the region of smaller values of b, and, as a result, is transformed into straight line $b=b(\xi,\pi)$ (see lines 5 and 6 in Fig. 1). Integrating set (2.11), we get the following result:

$$b(\xi,\tau)=\xi+h_0\cos\tau,$$
$$F(\xi,\tau)=-h_0\sin\tau(1-\xi)+(h_0\cos\tau+1)/\alpha, \tag{4.11}$$
$$\tau_2\leqslant\tau\leqslant\pi.$$

Hence, for large amplitudes of radiowave (4.1) the influence of the substrate produces a qualitative change in the behavior of the electromagnetic field in the superconductor in comparison with case (3.1). It is noteworthy that here the magnetic induction in each point of the superconducting plate decreases with time and never is frozen as happens for $h_0\leqslant 1$ and, also, in the case of bilateral excitation for arbitrary values of the amplitude of the radiowave.

We shall present formulae which are necessary for the calculation of the real $R(h_0)$ and imaginary $X(h_0)$ parts of the surface impedance $Z=R-iX$ and, also, the absorptivity $A(h_0)$:

$$\frac{R(h_0)}{R(1)}=\frac{3}{h_0}\int_0^\pi d\tau F(0,\tau)\cos\tau,\quad R(1)=\frac{8\mu\omega d}{3c^2}; \tag{4.12}$$

$$\frac{X(h_0)}{X(1)}=\frac{4}{\pi h_0}\int_0^\pi d\tau F(0,\tau)\sin\tau,\quad X(1)=\frac{2\pi\mu\omega d}{c^2}, \tag{4.13}$$

$$\frac{A(h_0)}{A(1)}=\frac{3}{h_0}\int_0^\pi d\tau\left[F(0,\tau)\cos\tau-\frac{\alpha F^2(1,\tau)}{h_0}\right],$$
$$A(1)=\frac{c}{\pi}R(1)=\frac{8\mu\omega d}{3\pi c}. \tag{4.14}$$

The analysis shows that in the transition from small (3.1) to large (4.1) values of the amplitude of the radiowave, i.e. at point $h_0=1$, functions $R(h_0)/R(1)$, $A(h_0)/A(1)$ and $X(h_0)/X(1)$ are continuous. At the same time their first derivatives at this point undergo a jump for every finite value of parameter α. This already indicates the existence of the size effect at the unilateral excitation of a superconductor which overlies a substrate with arbitrary dielectric properties. Below we shall discuss in detail about the influence of the substrate upon the size effect.

According to Eqs. (4.12), (4.13), (4.14) and (4.9) quantities $R(h_0)/R(1)$, $X(h_0)/X(1)$ and $A(h_0)/A(1)$ at large amplitudes (4.1) are functionals of dimensionless induction $b(1,\tau)$ at the interface $\xi=1$. In the second stage of the time evolution of fields the quantity $b(1,\tau)$ is found by solving nonlinear equation (4.7). This solution can be analyzed only for small and large values of parameter α. For this reason in the following we shall solve analytically our problem only for the cases of optically slow $\alpha\gg 1$ and fast $\alpha\ll 1$ substrate. Results for arbitrary values of parameter α will be obtained by numerical calculations.

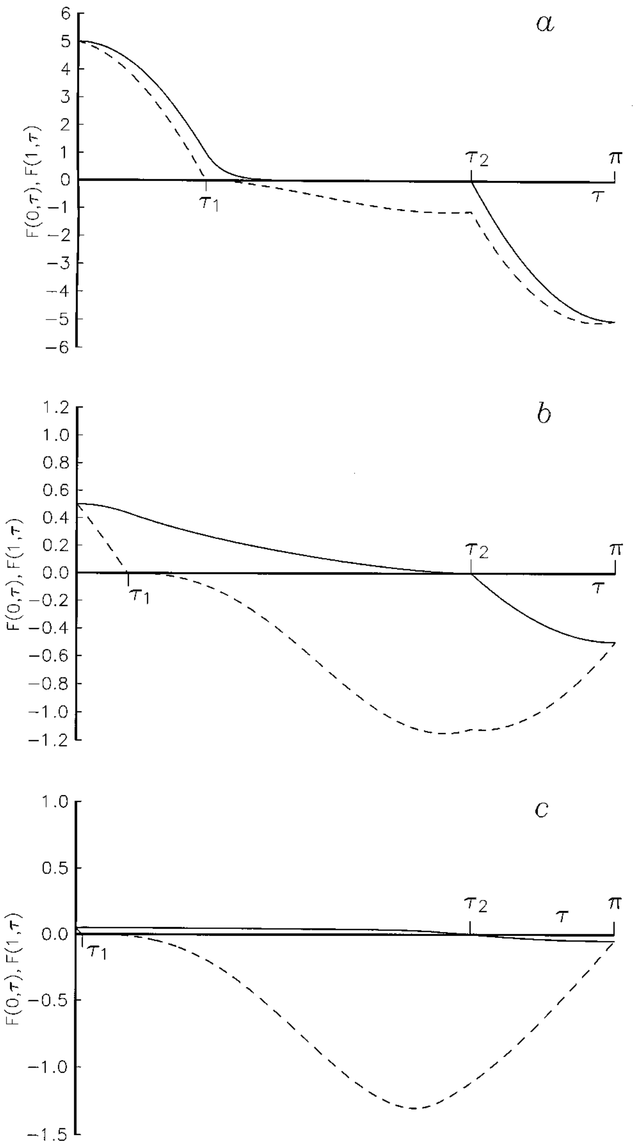

FIG. 2. Numerically calculated temporal dependences of dimensionless electric field at the free surface, $F(0,\tau)$ (dashed line), and the interface plate-substrate, $F(1,\tau)$ (solid line), for $h_0=1.5$ and $\alpha=0.1$ (a), 1 (b), 10 (c).

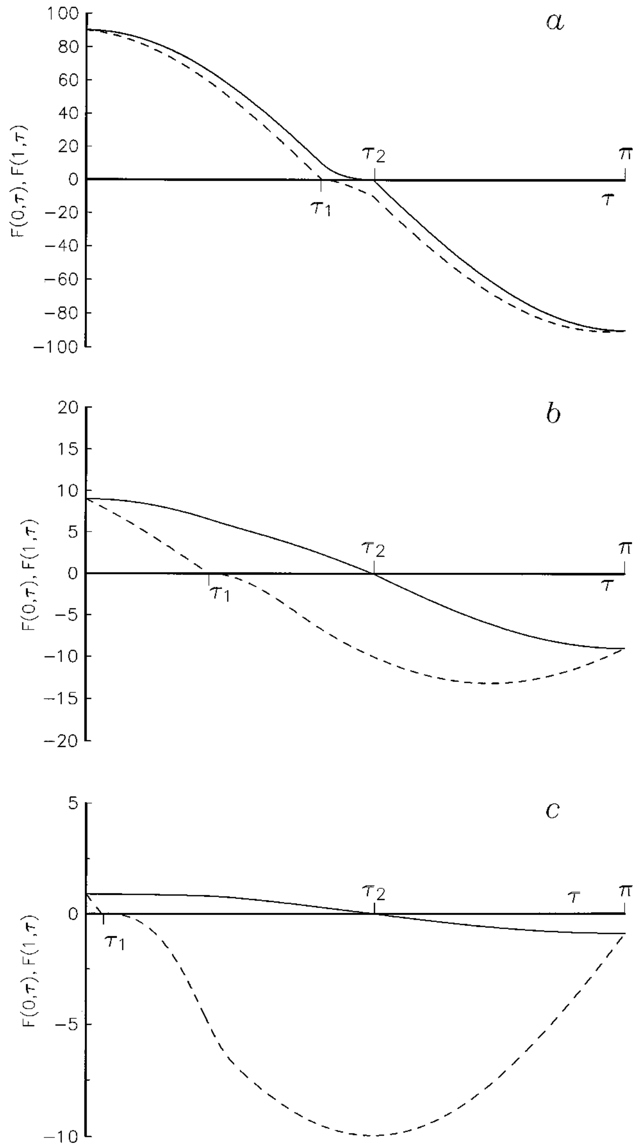

FIG. 3. Numerically calculated temporal dependences of dimensionless electric field at the free surface, $F(0,\tau)$ (dashed line), and the interface plate-substrate, $F(1,\tau)$ (solid line), for $h_0=10$ and $\alpha=0.1$ (a), 1 (b), 10 (c).

A. Optically slow substrate

Let us analyze the case

$$\alpha \gg 1 \quad \text{i.e.,} \ (c/\mu\omega d)^2 \ll \varepsilon. \tag{4.15}$$

Firstly, we should investigate what happens with each of the three described stages of the evolution of $b(\xi,\tau)$. According to Eq. (4.4), with $\alpha\to\infty$ the time-point τ_1 between the first and second stages tends to zero ($\tau_1 \approx (1-h_0^{-1})/\alpha$). This means that in the case of optically slow substrate (4.15) the first stage (4.3) practically is absent and the second one begins from $\tau=0$.

The first term in the equation (4.7) for induction $b(1,\tau)$ can be neglected when inequality (4.15) holds. In this case the solution of Eq. (4.7) have different form in two distinct lapses of the second stage. First (at $0\leqslant\tau\leqslant\tau_l$), the derivative $\partial b(1,\tau)/\partial\tau=0$. Here, in accordance with the initial condition Eq. (4.8), and also Eqs. (4.5), (4.9), the dimensionless induction at the interface and the electric field at the free boundary are:

$$b(1,\tau)=h_0-1;$$

$$F(0,\tau)=-h_0^2\sin^2(\tau/2)\sin\tau; \tag{4.16}$$

$$0\leqslant\tau\leqslant\tau_l.$$

At time-moment $\tau=\tau_l$ the derivative $\partial b(1,\tau)/\partial\tau=0$, and the expression in square brackets of Eq. (4.7) also vanishes. Hence, the equation for τ_l is gotten:

$$\cos\tau_l=1-2h_0^{-1}. \tag{4.17}$$

Furthermore, at $\tau_l<\tau\leqslant\tau_2$, quantity $\partial b(1,\tau)/\partial\tau$ is different from zero, and the expression in square brackets of Eq. (4.7) is equal to zero. Therefore,

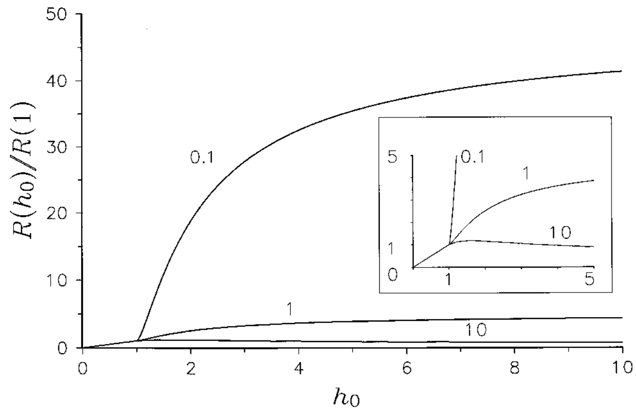

FIG. 4. Dependences $R(h_0)/R(1)$ at different values of α indicated near curves.

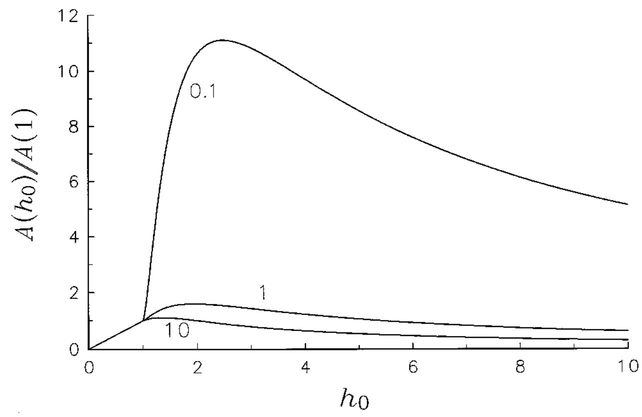

FIG. 6. Dependences $A(h_0)/A(1)$ at different values of α indicated near curves.

$$b(1,\tau)=1+h_0\cos\tau;$$
$$F(0,\tau)=-h_0\sin\tau; \quad (4.18)$$
$$\tau_l\leqslant\tau\leqslant\pi.$$

Fields $b(1,\tau)$ and $F(0,\tau)$ have the same asymptotics in the third stage of their evolution (see Eq. (4.11)). For this reason, we extended the domain of definition of Eq. (4.18) up to the end of the half-period.

In conclusion, in the case of large amplitudes of the radiowave (4.1) and optically slow substrate (4.15) the three stages characterizing the variation of the electromagnetic field in the superconductor become only two.

After substituting Eqs. (4.16) and (4.18) into expressions (4.12), (4.13), (4.14) and doing some necessary calculations, we get for the case of optically slow substrate (4.15):

$$\frac{R(h_0)}{R(1)}=\frac{A(h_0)}{A(1)}=\frac{3}{h_0}-\frac{2}{h_0^2}; \quad (4.19)$$

$$\frac{X(h_0)}{X(1)}=2-\frac{2-h_0}{\pi}\arccos\left(1-\frac{2}{h_0}\right)+\frac{4}{\pi}\left(1-\frac{2}{h_0}\right)^2(h_0-1)^{1/2}-\frac{16}{3\pi h_0^2}(h_0-1)^{3/2}. \quad (4.20)$$

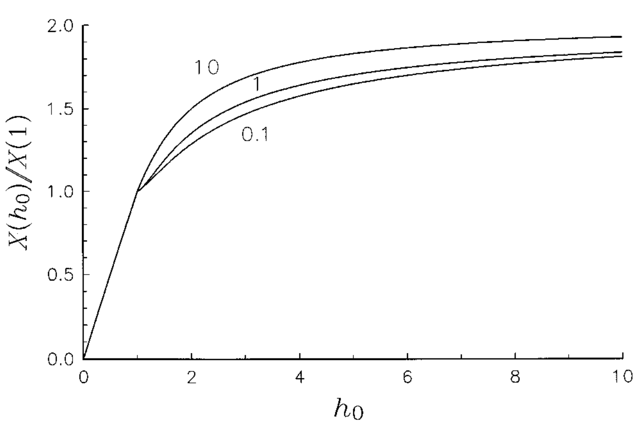

FIG. 5. Plots of $X(h_0)/X(1)$ at different values of α indicated near curves.

Note that results for large α do not depend on the value of the dielectric constant (i.e., on the properties of the optically slow substrate). Moreover, formulas for the impedance and the absorptivity turn out to be the same as for bilateral excitation. This is clear, since with $\alpha\gg1$ practically all electromagnetic energy, that comes from the superconductor to the substrate, is reflected from the interface. The reflected wave from the substrate plays the role of a second wave, which at bilateral excitation irradiates the superconductor from boundary $x=d$. Therefore, in the case of an optically slow substrate (4.15) a manifest size effect takes place: from Eqs. (4.19) and (4.20) it follows that the real part of the impedance and the absorptivity reach a maximum at $h_0=4/3$, and the imaginary part of Z has a steplike change at the same value of h_0.

The formulas obtained in the present Subsection can be applied in the situation when, for instance, a good conductor serves as a substrate inside which the electric field is assumed to be zero (i.e., $F(d,t)=0$, $\varepsilon\to\infty$).

B. Optically fast substrate

Now, let us consider the situation when the superconducting plate overlies a substrate with small dielectric constant,

$$\alpha\ll1 \quad \text{i.e.,} \quad 1\leqslant\varepsilon\ll(c/\mu\omega d)^2. \quad (4.21)$$

During the calculation of the asymptotics for the impedance and absorptivity we have restricted ourselves to the linear approximation in parameter α. For this reason, in the first and third stages of the time evolution of the fields it is necessary to use exact expressions (4.3), (4.11) for induction $b(\xi,\tau)$ at the interface $\xi=1$ and electric field $F(\xi,\tau)$ at the free boundary $\xi=0$. In the second stage (4.5) dimensionless induction $b(1,\tau)$ is so small that it can be considered equal to zero. Indeed, according to Eqs. (4.7), (4.8) quantity $b(1,\tau)$ at the beginning of the second stage is proportional to α and thereafter decreases, since derivative $\partial b(1,\tau)/\partial\tau$ is negative in the whole second stage. The decrease of function $b(1,\tau)$ has an exponential behavior with a characteristic scale of the order of α. Then, in the second stage the terms with $b(1,\tau)$ in formulae (4.12), (4.13), (4.14), are of the order α^3 and can be omitted.

For small α the expressions for the surface impedance and the absorptivity, which are valid in the whole region $h_0 \geqslant 1$, are quite cumbersome. At the same time, for analytic investigations of these functions it is enough to study only the region $\alpha^2 \ll h_0^2 - 1$, excluding the narrow vicinity of point $h_0 = 1$. This allows to solve equation (4.4) in the linear approximation in α and to find the explicit form for τ_1. From the preceding, in the case of optically fast substrate (4.21) we get

$$\frac{R(h_0)}{R(1)} = \frac{3}{\alpha}\left[\arccos(h_0^{-1}) - \frac{(h_0^2-1)^{1/2}}{h_0^2} + \frac{\alpha}{3h_0^2}\right]; \qquad (4.22)$$

$$\frac{X(h_0)}{X(1)} = 1 + \frac{2}{\pi}\arccos(h_0^{-1}) - \frac{2}{\pi}\frac{(h_0^2-1)^{1/2}}{h_0^2}; \qquad (4.23)$$

$$\frac{A(h_0)}{A(1)} = \frac{1}{\alpha h_0^2}[6(h_0^2-1)^{1/2} - 6\arccos(h_0^{-1}) + \alpha]. \qquad (4.24)$$

The analysis of formulas (4.22), (4.23) and (4.24) shows, that even in the case (4.21), when the substrate has a small ε, the size effect manifests itself in the response of the superconducting plate to an electromagnetic wave. With increasing dimensionless amplitude h_0, the imaginary part of the impedance increases monotonously from the value $X(1)$ at $h_0 = 1$ up to $2X(1)$ at $h_0 \geqslant 1$. In addition its derivative with respect to h_0 has a steplike change in the narrow vicinity (with width of order α) of point $h_0 = 1$. A considerable size effect appears in the real part of the impedance which increases $\alpha^{-1} \geqslant 1$ times in comparison with $R(1)$. However, absorptivity undergoes the more evident changes. It has a maximum with relative magnitude $A^{(max)}/A(1) \sim \alpha^{-1}$ at point $h_0 = h_m$ that is found from equation

$$(h_m^2 - 1)^{1/2} = 2\arccos(h_m^{-1}). \qquad (4.25)$$

V. NUMERICAL RESULTS

In Figs. 2, 3 we present numerically calculated graphs for the dimensionless electric field $F(\xi,\tau)$ at $\xi = 0$ and $\xi = 1$. During the calculation we used formulas (4.3)–(4.11). Curves of Figs. 2 and 3 were obtained for $h_0 = 1.5$ and $h_0 = 10$, respectively, and illustrate the effect of parameter α (i.e., dielectric constant ε, see (2.11)) on the time dependences of the electric field at both plate boundaries. In each graph we have indicated the time interval (τ_1, τ_2) in which the spatial distribution of the electric field inside the superconducting plate is sign variable.

It is noteworthy that for arbitrary values of α and $h_0 > 1$ fields $F(0,\tau)$ and $F(1,\tau)$ have the same magnitude at time moments $\tau = 0, \pi$ ($F(0,0) = F(1,0)$, $F(0,\pi) = F(1,\pi)$, see Figs. 2, 3). This fact can be easily verified with Eqs. (4.3), (4.11). For the case $\alpha = 0.1$ (Figs. 2a, 3a), corresponding to an optically fast substrate, functions $F(0,\tau)$ and $F(1,\tau)$ have similar behaviors. This similitude is better for large amplitudes of the ac signal ($h_0 = H_0/H_p \geqslant 1$). As the optical density (i.e., ε) is increased (curves b,c in Figs. 2, 3), $\tau_1 \to 0$ and functions $F(0,\tau)$, $F(1,\tau)$ become quite different one from the other because the electric field at the interface superconductor-substrate decreases with α ($F(1,\tau) = b(1,\tau)/\alpha$) whereas $F(0,\tau)$ acquires the asymptotic form given by Eqs. (4.16), (4.18).

The calculation of the electric field at plate boundaries allows to find the surface impedance $Z = R - iX$ Eqs. (4.12), (4.13) and the absorptivity A (4.14). Figs. 4–6 show numerically calculated plots for $R(h_0)$, $X(h_0)$ and $A(h_0)$ for the same values of α as in Figs. 2, 3. From graphs it is evident that the manifestation of the size effect depends considerably on the substrate. Thus, as α is diminished $R(h_0)$ (Fig. 4) undergoes an abrupt increase at $h_0 \sim 1$ and its maximum, which is observed in the case of an optically slow substrate at $h_0 \sim 4/3$, disappears. Notice (Fig. 5) that for small values of α the curve $X(h_0)$ has a break near $h_0 \sim 1$. The absorptivity $A(h_0)$ (Fig. 6), which exhibits a maximum for any value of the dielectric constant of the substrate, is notably increased by lowering parameter α. So, the numerical results for $\alpha = 0.1$ and $\alpha = 10$ confirm the analytical ones obtained in Secs. IV A, IV B for the limiting cases $\alpha \ll 1$ and $\alpha \geqslant 1$.

In conclusion, the ac response of hard superconductors to an incident plane wave is notably affected by the presence of the substrate. The peculiarities of the surface impedance and absorptivity associated with the size effect turn out to be strongly correlated with the value of the dielectric constant of the substrate. Our analysis is based on the critical state model, however, similar results can be obtained by using other approaches.

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