# Selective transparency of single-mode waveguides with surface scattering

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Random surface scattering in a one-mode waveguide is studied for a surface profile that has long-range correlations along the waveguide. Analytical treatment of this scattering shows that, with the proper choice of surface, one can arrange any desired combination of transparent and nontransparent frequency windows. We suggest a method for finding such profiles and demonstrate its effectiveness by making use of direct numerical simulations. © 2001 Optical Society of America

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Recently much attention has been paid to one-dimensional (1D) solid-state models with correlated disorder. The interest in this subject is due to results that demonstrate a direct relevance of anomalous transport properties to particular pair correlations in scattering potential. Specifically, it was shown[1–3] that any desired combination of transparent and nontransparent frequency intervals can be achieved by proper construction of potentials with long-range correlations. Experimental realization of such potentials for single-mode waveguides with deltalike scatters inserted[4] has proved that it is possible to construct devices that are transparent for any given range of frequency.

There is a well-developed theory of wave propagation through surface-disordered waveguides (see, e.g., Refs. 5–8 and references therein). The subject of wave propagation is important both from the theoretical viewpoint and for experimental applications such as for optical fibers, remote sensing, radio wave propagation, and shallow water waves.[5,6] One should note that the main results in this field are for random surfaces with fast-decaying correlations along a scattering surface. It is now of great interest to explore the role of slowly decaying correlations that are expected to result in anomalous surface-scattering properties.

We stress that the correspondence between surface and bulk scattering still deserves detailed study. In this respect, we mention recent results[7,8] in which specific properties of surface scattering were discovered that are different from those known in standard quasi-1D models with random potentials.[9]

In this Letter we analyze surface scattering for the case of one open channel, assuming long-range correlations in the surface potential. Our main interest is in exploring the possibility of constructing such surfaces that result in windows whose transparency is related to the frequency of incoming waves.

We consider a two-dimensional waveguide of length $L$ along the $x$ axis, with a flat upper surface $z = d$ and a rough (corrugated) lower surface $z = \xi(x)$; see also Refs. 8, 10, and 11. Dirichlet boundary conditions on both walls of the waveguide are assumed. Random function $\xi(x)$ is characterized as follows:

$$\langle \xi(x) \rangle = 0, \qquad \langle \xi(x)\xi(x') \rangle = \sigma^2 \mathcal{W}(|x - x'|). \tag{1}$$

The angle brackets stand for a statistical average over the ensemble of realizations of $\xi(x)$, and the root mean square $\sigma$ determines the roughness strength. Binary correlator $\mathcal{W}(|x|)$ decreases with a typical scale $R_c$, which is of the order of a mean length of surface irregularities. Obviously, a statistical treatment of the surface scattering is meaningful only if correlation length $R_c$ is much less than length size $L$ of a waveguide, $R_c \ll L$. In addition, we assume that the roughness is weak, $\sigma \ll d$.

Keeping in mind the relevance of surface scattering to the Anderson localization in our model, we consider in what follows a single-mode waveguide when there is only a propagating mode with a real longitudinal wave number $k_1 = [(\omega/c)^2 - (\pi/d)^2]^{1/2}$. All other modes are evanescent; therefore, width $d$ is restricted by the conditions $0 < k_1 d/\pi < \sqrt{3}$. Note that the assumed condition $\sigma \ll d$ leads to the inequality $k_1 \sigma \ll 1$.

It can be shown[10,11] that the corresponding wave equation takes the form

$$\left[\frac{d^2}{dt^2} + (k_1 R_c)^2\right]\Psi(t) = \frac{2}{\pi}\frac{\sigma}{R_c}\left(\frac{\pi R_c}{d}\right)^3 \varphi(t)\Psi(t), \tag{2}$$

with $t = x/R_c$. Here the function $\varphi(t)$ is determined by the relation $\xi(x) = \sigma\varphi(x/R_c)$. One can see that surface scattering in a single-mode waveguide is reduced to a 1D model with random potential $\varphi(t)$. Therefore its solution is entirely consistent with the theory of 1D localization. In accordance with the form of the perturbation potential, localization length $L_{\text{loc}}$ is given by the following expression[10,11]:

$$L_{\text{loc}}^{-1} = \frac{\sigma^2}{\pi^2}\left(\frac{\pi}{d}\right)^6 \frac{W(2k_1)}{(2k_1)^2}, \tag{3}$$

where $W(k_x)$ is the Fourier transform of binary correlator $\mathcal{W}(|x|)$ and $\mathcal{W}(|x|) = \int_{-\infty}^{\infty} \mathrm{d}k_x/2\pi \exp(ik_x x)W(k_x)$. Note that $W(k_x)$ is a positive function of the order of $R_c$, which decreases depending on $|k_x|$ with a typical scale ${R_c}^{-1}$.

Equation (3) for localization length $L_{\mathrm{loc}}$ gives complete information about transmission through the waveguide. In particular, it determines the transmittance and its fluctuations for any ratio $L/L_{\mathrm{loc}}$; see, e.g., Refs. 10–12. As one can see, the binary correlator of the surface profile defines all transport properties of a one-mode waveguide. In particular, if $W(2k_1)$ vanishes within some interval of wave number $k_1$, the waveguide is entirely transparent. Below, we show how to construct surface profiles that result in complete transparency in a given range of $k_1$.

To this end, we generalize the approach[3] developed for the Kronig–Penney model with the correlated disorder. This model can be treated as a particular case of Eq. (2) when the function $\varphi(t)$ is given in the form of periodic delta kicks with random amplitudes.

In the case of continuous potential we can represent $\varphi(t)$ in the following form:

$$\varphi(t) = \int_{-\infty}^{\infty} \mathrm{d}t' Z(t - t')\beta(t'), \tag{4}$$

with some function $\beta(t)$, which is entirely determined by Fourier transform $W(k_x)$ of the binary correlator $\mathcal{W}(|x|)$,

$$\beta(t) = \sqrt{R_c} \int_{-\infty}^{\infty} \frac{\mathrm{d}k_x}{2\pi} \exp(ik_x R_c t)W^{1/2}(k_x). \tag{5}$$

Here $Z(t)$ is a dimensionless delta-correlated random process (white noise) with $\langle Z(t)\rangle = 0$ and $\langle Z(t)Z(t_0)\rangle = \delta(t - t_0)$.

Equations (4) and (5) allow us to solve practically the inverse scattering problem of constructing a potential from its binary correlator. Note that this construction is possible in the case of a weak disorder, $\sigma \ll d$, therefore only a binary correlator is involved in the reconstruction of $\varphi(t)$. For this reason, the solution above is not unique because higher correlators are not controlled. Below, we demonstrate the suggested approach by considering two simple cases of a correlated surface profile.

Let us first consider the case when the waveguide is completely transparent for $k_1 > 1/2R_c$. In this case one can get the following expressions for the binary correlator and its Fourier transform:

$$\mathcal{W}_1(|x|) = \frac{\sin(x/R_c)}{x/R_c}, \tag{6}$$

$$W_1(k_x) = \pi R_c \Theta(1 - |k_x|R_c), \tag{7}$$

with $\Theta(x)$ as the unit-step function, $\Theta(0) = 1/2$. This kind of $\Theta$-like dependence of $W(k_x)$ was recently analyzed in Refs. 13 and 14. The surface profile that has these correlations is described by the function

$$\varphi_1(t) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \mathrm{d}t' Z(t - t')(\sin t'/t'). \tag{8}$$

In this case the inverse localization length is

$$\frac{1}{L_{\mathrm{loc1}}(k_1)} = \frac{1}{\pi R_c}\left(\frac{\sigma}{R_c}\right)^2 \left(\frac{\pi R_c}{d}\right)^6 \frac{\Theta(1 - 2k_1 R_c)}{(2k_1 R_c)^2}. \tag{9}$$

Therefore, with an increase in wave number $k_1$, the localization length increases smoothly and then goes abruptly to infinity for $k_1 > 1/2R_c$. Such behavior can be observed if the transition point $k_1 = 1/2R_c$ is located inside an allowed single-mode region, i.e., for $12(\pi R_c/d)^2 > 1$. To observe this effect for finite waveguides, one needs to assume that at the transition point the regime of a strong localization holds:

$$\frac{L}{L_{\mathrm{loc1}}(1/2R_c)} \equiv \frac{1}{2\pi}\left(\frac{\sigma}{R_c}\right)^2 \left(\frac{\pi R_c}{d}\right)^6 \frac{L}{R_c} \gg 1. \tag{10}$$

In this case the average transmittance is expected to be exponentially small owing to a strong localization for $k_1 < 1/2R_c$. Otherwise, in the interval $1 < (2k_1R_c)^2 < 12(\pi R_c/d)^2$, a ballistic regime occurs with perfect transparency.

The second case refers to a complimentary situation when for $k_1 < 1/2R_c$ the waveguide is transparent and for $k_1 > 1/2R_c$ it is not. The corresponding expressions for $\mathcal{W}(|x|)$ and $W(k_x)$ are

$$\mathcal{W}_2(|x|) = \pi\delta(x/R_c) - \frac{\sin(x/R_c)}{x/R_c}, \tag{11}$$

$$W_2(k_x) = \pi R_c \Theta(|k_x|R_c - 1). \tag{12}$$

In this case the corrugated surface is described by a superposition of white noise and roughness of the first type. As a result, the surface-profile potential $\varphi(t)$ takes the form

$$\varphi_2(t) = \sqrt{\pi}\, Z(t) - \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \mathrm{d}t' Z(t - t')(\sin t'/t'). \tag{13}$$

Correspondingly, the inverse localization length is expressed by

$$\frac{1}{L_{\mathrm{loc2}}(k_1)} = \frac{1}{\pi R_c}\left(\frac{\sigma}{R_c}\right)^2 \left(\frac{\pi R_c}{d}\right)^6 \frac{\Theta(2k_1 R_c - 1)}{(2k_1 R_c)^2}. \tag{14}$$

Therefore, in contrast to the first case, here the localization length $L_{\mathrm{loc2}}(k_1)$ is equal to infinity below transition point $k_1 = 1/2R_c$. With a further increase of $k_1$, it increases smoothly starting from the value $L_{\mathrm{loc2}}(1/2R_c)$. Again, transition point $k_1 = 1/2R_c$ is assumed to be inside the single-mode interval. In addition, we assume that a strong localization is retained at upper point $k_1 = \pi\sqrt{3}/d$ of the single-mode region, i.e.,

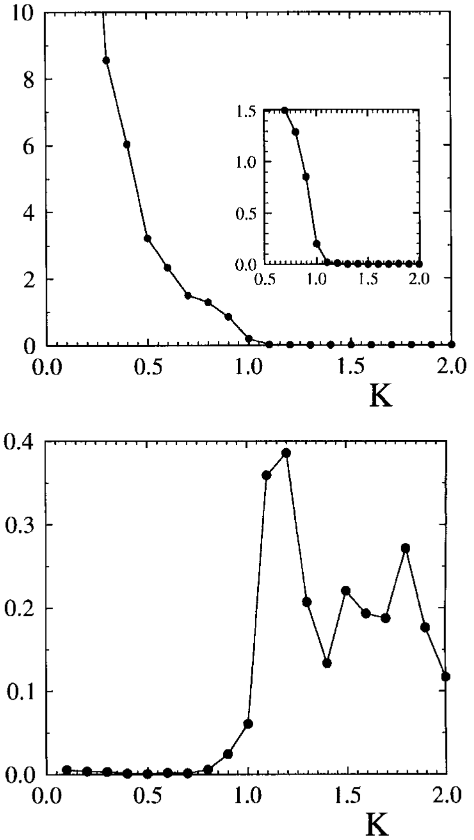

Fig. 1. Selective dependence of the rescaled localization length on the normalized wave vector for two realizations of a random surface with long-range correlations. Inset, enlarged view of the transition region about point K = 1.

$$\frac{L}{L_{\mathrm{loc}2}(\pi\sqrt{3}/d)} \equiv \frac{1}{12\pi}\left(\frac{\sigma}{R_c}\right)^2\left(\frac{\pi R_c}{d}\right)^4\frac{L}{R_c} \gg 1\cdot \qquad (15)$$

In this case the ballistic transparency is abruptly replaced by a strong localization at transition point $k_1 = 1/2R_c$.

Let us now demonstrate the above predictions by direct numerical simulations. We compute $L_{\mathrm{loc}}$ by the method that is valid for arbitrary surface roughness and is described in Refs. 1 and 2. We approximate the continuous function $\varphi(t)$ in Eq. (2) by the sum of delta kicks with the spacing $\delta$ chosen much smaller than any physical length scale in our model. In this way one can write a Hamiltonian map and use it to find the localization length by means of the Lyapunov exponent $\lambda = L_{\mathrm{loc}}{}^{-1}$ of a dynamic problem associated with this map (see details in Refs. 1–3).

For the numerical data shown in Fig. 1, the dimensionless Lyapunov exponent $\Lambda = c_0\lambda$ is plotted against the normalized wave vector $K = 2k_1R_c$ for the range $0 < K < 2$, which corresponds to the single-mode interval. Coefficient $c_0$ permits $\Lambda = K^{-2}$ for the delta-correlated potential. One can clearly see the expected nontrivial dependence of $\Lambda$ on the wave vector, which is due to specific binary correlations in the potential $\varphi(t)$. The fluctuations of $\Lambda$ are due to numerical restrictions (finite length of the waveguide, finiteness of spacing $\delta$, and nonvanishing variance of the potential).

Potential $\varphi(t)$ was constructed according to discrete versions of Eqs. (8) and (13) that determine complimentary dependence of localization length $L_{\mathrm{loc}}(k_1)$; see Eqs. (9) and (14). The data demonstrate the strong dependence of $\Lambda$ on the wave vector at the crossing of the transition point $K = 1$. By taking size $L$ in accordance with Eq. (10) and inequality (15), one can arrange the selective transparency of waveguides as predicted by theory.

In conclusion, we have studied the possibility of constructing one-mode waveguides with selective transparency that depends on the wave vector of an incoming wave. An analytical treatment showed that this can be done by proper choice of random surfaces with specific long-range correlations along waveguides. Numerical data for two cases with complimentary dependences of the localization length on the wave vector demonstrate the effectiveness of the theoretical predictions. The results presented here may be used for experimental realizations of waveguides with desired selectivity of transmission. Note that random surfaces with discontinuous dependence of $W(k_x)$ were recently fabricated in an experimental study of backscattering enhancement.[15]

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