Electron localization in narrow surface-corrugated conducting channels: Manifestation of competing scattering mechanisms

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Transport properties of narrow two-dimensional conducting wires in which the electron scattering is caused by side edges roughness have been studied. A method for calculating dynamic characteristics of such conductors is proposed which is based on a two-scale representation of the mode wave functions at weak scattering. With this method, fundamentally different *by-height* and *by-slope* scattering mechanisms associated with edge roughness are discriminated. The results for single-mode systems, previously obtained by conventional methods, are proven to correspond to the former mechanism only. Yet the commonly ignored by-slope scattering is more likely dominant. The electron extinction lengths relevant to this scattering differ substantially in functional structure from those pertinent to the by-height scattering. The transmittance of ultraquantum wires is calculated over all range of scattering parameters, from ballistic to localized transport of quasiparticles. The obtained dependence of scattering lengths on the disorder parameters is qualitatively valid for an arbitrary intercorrelation of the boundaries' defects.

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I. INTRODUCTION

The influence of different-types of inhomogeneities and defects on transmission properties of waveguides for both quantum and classical waves is a subject of long-term intensive research. Most theoretical studies have been restricted to flat surfaces. In practice, however, side boundaries of mesoscopic conductors are either inherently rough (due to growth, fracture, etc.), or artificially patterned (e.g., due to lithographic preparation). One way or the other, surface-corrugated conducting systems find widespread applications in technology and material science (quantum conductors, optical fibers, etc.), and, hence, it is often necessary to know how the shape of the confining surface affects the charge or classical wave transport in different systems.

Of peculiar interest for contemporary microelectronics, from both applied and theoretical viewpoints, are striplike two-dimensional (2D) conductors of mesoscopic size. Nowadays there exists widely established opinion (see, e.g., Refs. 1–13 and references therein) that dynamic properties of pure-in-bulk quantum wires, in particular two dimensional, are largely determined by scattering the electrons from randomly rough side boundaries of the conductor. This scattering mechanism is proven to be responsible for both relaxation processes in multimode conductors[8–10] and for nondissipative (Anderson) localization of conduction electrons in narrow single-mode wires.[2,3,11]

In studying the electron transport in surface-corrugated systems two main problems are especially highlighted. One of them is relevant to adequate description of the electron scattering from statistically rough surfaces. The other, dynamic, problem is pertinent to a proper consideration of the interference of multiply scattered quantum waves, which is essential for describing, within the framework of a perturbation theory, the effects resulting from nondissipative localization of the electron states.

To resolve the first problem and make an adequate comparison with experimental data, one needs a theory which relates transport properties of the conduction electrons to the shape of bounding surfaces of the conductor. Since in random-inhomogeneous flat wires fluctuations of both side boundaries can be considered as either mutually independent or correlative, subject to the preparation technology, it is important to trace the relation between the kinetic quantities and the statistics of boundary irregularities, as well as mutual correlation of the opposite boundaries.

In a previous paper[11] we studied the case of a single-mode 2D conductor with statistically identical rough side boundaries. The particular model considered in Ref. 11 of a wire with completely correlated boundaries (CCB's) is equivalent to the deterministic waveguide system of constant width whose inhomogeneities consist in solely waveguide bends. It was shown that the electron dynamics in such conducting strips is governed by quite a different Hamiltonian than that pertinent to the seemingly more general model of a 2D conductor with one boundary which is rough and the other ideally flat.[2] A substantial distinction of the Hamiltonians in Refs. 2 and 11 have resulted in a qualitative functional distinction of the obtained scattering lengths.

The results of Refs. 2 and 11 provided the grounds to propose that in those works the quantum wave scattering can be assumed to be associated with fundamentally different physical factors, namely, with a deviation of the boundaries from their ''ideal'' shape in Ref. 2 and a fluctuation of their slopes in Ref. 11. Provided the supposition is true, when studying particle or classical wave transport in surface-corrugated waveguide systems one has to distinguish between two different noninterfering scattering mechanisms which we call by-height (BH) and by-slope (BS) scattering. However, it should be noted that quantum wave scattering was analyzed in Refs. 2 and 11 for different waveguide geometries and with the use of substantially different methods.

Therefore, the conjecture stated in Ref. 11 about the relative significance of these scattering mechanisms needs to be additionally substantiated.

To avoid possible misunderstanding and support our idea of different scattering mechanisms pertaining to the imperfect boundaries of guiding systems, we examine the waveguide (conductor) geometry admitting of both BH and BS scattering simultaneously. We consider a 2D conducting strip with statistically symmetrical rough boundaries (the abbreviation SSB will be used for such a strip, in contrast to the CCB strip considered in Ref. 11), which is physically equivalent to a waveguide with a straight central line (guiding axis) and a randomly fluctuating width. It will be shown below that the suggested model of the waveguide corresponds qualitatively to a 2D wire with opposite side boundaries whose intercorrelation can be thought of as arbitrary. For the SSB model, as well as for an arbitrary 2D waveguide, the presence is typical of both BH and BS scattering mechanisms. It is noteworthy that these mechanisms compete with one another, depending on the statistical roughness parameters, even in the simplest case of boundary asperities being small in height and rather smooth.

Technically, specifying the scattering mechanisms is made most straightforward by reducing the problem of the electron scattering from complicated boundaries of the conductor to the appropriate "bulk" problem specified by complex Hamiltonian but simple boundary conditions. An analysis of the problem thus formulated can often be found much easier than an analysis of the problem with complicated boundary conditions. In some cases it can even be performed nonperturbatively. In this work, using such an approach, we managed to reasonably discriminate between BH and BS scattering mechanisms, and analyzed their competition in the electron-surface scattering. In addition, solving the "surface" problem in a "bulk" formulation enabled us to carefully trace such a fine *spectral* effect as the Anderson localization of current carriers.

Note that the method for solving the problems of the by-surface scattering through reduction to the Hamiltonian form is not quite original. It was employed for a long time in theories of classical and quantum wave scattering, in particular in Refs. 14, 8, and 10, where nonlinear coordinate transformation was used, which smooths out the rough surface to the flat one. In this paper, an analogous reduction of a "surface" problem to the Hamiltonian formulation is made by merely going over to the *local mode representation*. This approach ensures the optimal choice of trial quantum states which serve as a basis for the perturbation theory. In our view, it is essential that the mode states *a priori* contain information on the lateral confinement of the system under consideration and, therefore, are more adjusted to a perturbative treatment of transport problems in waveguidelike systems than the widely used isotropic plane-wave basis.

II. STATEMENT OF THE PROBLEM: CHOOSING STATISTICAL MODEL

We consider a conducting strip of average width D with the nonuniform stretch of length L (shaded region in Fig. 1)

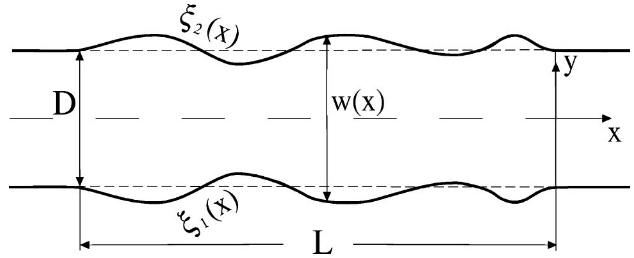

FIG. 1. 2D electron waveguide with rough side boundaries pertaining to the SSB class.

occupying [in the (x,y) plane] the area restricted by the inequalities

$$\begin{gathered} -L/2 \leqslant x \leqslant L/2, \\ -D/2+\xi_1(x) \leqslant y \leqslant D/2+\xi_2(x). \end{gathered} \tag{2.1}$$

Functions $\xi_{1,2}(x)$ describing the side boundaries' roughness are assumed to be random processes with zero mean values. It turns out to be more convenient for subsequent analysis to introduce, instead of $\xi_{1,2}(x)$, two different random functions, viz.

$$w(x)=D+\Delta\xi(x) \quad \text{and} \quad \xi_c(x)=[\xi_1(x)+\xi_2(x)]/2, \tag{2.2}$$

where $\Delta\xi(x)=\xi_2(x)-\xi_1(x)$. Function $w(x)$ is nothing but the fluctuating width of the strip, whereas $\xi_c(x)$ describes the y coordinate of the randomly fluctuating central (symmetry) line of the 2D waveguide. In terms of functions (2.2), the disordered region [Eq. (2.1)] is represented by inequalities

$$\begin{gathered} -L/2 \leqslant x \leqslant L/2, \\ -w(x)/2+\xi_c(x) \leqslant y \leqslant w(x)/2+\xi_c(x). \end{gathered} \tag{2.3}$$

A reformulation of the problem in terms of the characteristic functions is advantageous from a mathematical point of view. In addition, it gives one a clear idea of the part played by different physical factors in quantum wave scattering in surface-corrugated conductors.

Bearing in mind the statistical nature of the problem under consideration, it is necessary to specify correlation properties of random variables (2.2) in conformity with the conductor geometry chosen. The mean values of functions $w(x)$ and $\xi_c(x)$ are naturally $\langle w(x)\rangle = D$ and $\langle \xi_c(x)\rangle = 0$; the angular brackets stand for statistical averaging over realizations of $\xi_{1,2}(x)$. Unlike the plain averages, the binary correlators of functions (2.2) are not so uniquely predetermined, but depend significantly on the intercorrelation between the side boundaries of the conducting strip. Here we consider a correlation model in whose frames both of the boundaries are regarded statistically identical in that the functions $\xi_{1,2}(x)$ obey the equalities

$$\langle \xi_i(x)\rangle = 0 \quad \text{and} \quad \langle \xi_i(x)\xi_i(x')\rangle = \sigma^2\mathcal{W}(x-x'), \quad i=1,2. \tag{2.4}$$

Here σ is the rms height of the asperities, which is thought to be identical for both of the strip edges, $\mathcal{W}(x)$ is the correlation coefficient specified by the unit maximal value and the correlation radius r_c.

As for the intercorrelation of the opposite-boundary roughness, two marginal options seem to be distinct among others for their particular symmetries. One of them, abbreviated in Ref. 11 as the CCB model, in terms of a statistical formulation, implies the fulfilment of correlation equality $\langle \xi_i(x)\xi_k(x')\rangle = \sigma^2\mathcal{W}(x-x')$ at $i\neq k$; it is physically equivalent to a 2D waveguide of constant width, whose inhomogeneities are in the form of random bends.

In this work, we make use of another particular model in which the displacements of the side boundaries comply with the first of relations (2.4), whereas for the opposite boundaries the correlation equality holds:

$$\langle \xi_i(x)\xi_k(x')\rangle = -\sigma^2\mathcal{W}(x-x'), \quad i\neq k. \tag{2.5}$$

It is easy to make sure that Eqs. (2.4) and (2.5) are equivalent to the following correlations:

$$\langle \xi_c(x)w(x')\rangle = 0, \tag{2.6a}$$

$$\langle \xi_c(x)\xi_c(x')\rangle = 0. \tag{2.6b}$$

Equality (2.6a) implies that the functions $\xi_c(x)$ and $w(x)$ can be thought of, within the correlation approximation, as statistically independent random processes. The second equality [Eq. (2.6b)], within the same approximation is consistent with a deterministic relation $\xi_1(x) = -\xi_2(x)$, that holds true in the case of the electron waveguide with side boundaries fluctuating symmetrically about the straight central line, as is shown in Fig. 1 (the SSB model).

From the linear response theory,[15] the dimensionless conductance $g(L)$ (in units of $e^2/\pi\hbar$) at $T=0$ is represented by the expression

$$g(L) = -\frac{4}{L^2}\int\int d\boldsymbol{r}\, d\boldsymbol{r}' \frac{\partial G(\boldsymbol{r},\boldsymbol{r}')}{\partial x}\frac{\partial G^*(\boldsymbol{r},\boldsymbol{r}')}{\partial x'}. \tag{2.7}$$

Here $G(\boldsymbol{r},\boldsymbol{r}')$ is the retarded one-electron Green function; integration with respect to $\boldsymbol{r}=(x,y)$ runs over the area [Eq. (2.3)] occupied by the irregular part of the wire. Within the isotropic Fermi-liquid model, the function $G(\boldsymbol{r},\boldsymbol{r}')$ is governed by the equation

$$(\Delta + k_F^2 + i0)G(\boldsymbol{r},\boldsymbol{r}') = \delta(\boldsymbol{r}-\boldsymbol{r}'), \tag{2.8}$$

where Δ is the two-dimensional Laplace operator, and k_F is the electron Fermi wave number. As to the boundary conditions to Eq. (2.8), in the direction of current (x) the conductor will be regarded as open whereas in the transverse direction (y) the zero (Dirichlet) conditions will be used. Note that formula (2.7) is valid asymptotically under conditions of *weak scattering* (WS),[16,17] which we assume in this paper [see Eq. (4.1) below].

The inhomogeneity of side boundaries of a waveguide system can be taken into account in a number of ways. The most commonly used method reduces to a linearization of the exact boundary conditions in small fluctuations of bounding surfaces, thus leading to a reformulation of the initial rough-surface problem as a problem with regular boundaries specified by randomly varying impedances.[18–20] However, such an approach can result, in some cases, in a significant underestimation of the intermode scattering, which often exerts the primary control over the dynamics of waves within bounded regions. It will be shown below that the entanglement of waveguide modes is governed by a fluctuation of the confining surfaces in *slope* rather than in their *displacement* from the specified ideal shape. Meanwhile, if one linearizes the boundary conditions, it becomes precisely the *height* of the surface roughness that serves as a determining factor for wave scattering. Therefore, an entanglement of the modes can either to a large extent or even entirely, be lost when expanding the boundary conditions in small heights of the surface roughness.

The intermode scattering can be taken into account using the methods of Refs. 21–23 or with the method of "smoothing" coordinate transformation, used in Refs. 8, 10, and 11. In the latter papers, the dynamic problem pertinent to a system with corrugated boundaries is reduced without any approximation to the analogous problem related to the system with ideal boundaries, though governed by a more complex Hamiltonian. This method is advantageous in that it enables one to analyze the dynamics of quasiparticles without resorting to the concept of "adiabaticity" of the confining potential.[24,25]

Technically, the procedure of "smoothing" rough boundaries of the conductor is most convenient to perform without applying explicitly the coordinate transformation, which, in addition, is quite nonlinear in general case. The same result can be obtained by going to the local (in lengthwise coordinate x) mode representation in Eqs. (2.7) and (2.8), i.e. by performing a Fourier transformation in the coordinate y using the complete set of "transverse" eigenfunctions of the Laplace operator which are consistent with boundary conditions prescribed at the true walls of the wire, $y=\pm w(x)/2 + \xi_c(x)$. In our case (Dirichlet conditions) we choose, for definiteness, the functions

$$S_n(y|x) = \left[\frac{2}{w(x)}\right]^{1/2} \sin\left[\left(\frac{y-\xi_c(x)}{w(x)} + \frac{1}{2}\right)\pi n\right], \quad n\in\aleph. \tag{2.9}$$

By substituting $G(\boldsymbol{r},\boldsymbol{r}')$ in the form of a double Fourier series,

$$G(\boldsymbol{r},\boldsymbol{r}') = \sum_{n,n'=1}^{\infty} S_n(y|x)G_{nn'}(x,x')S_{n'}(y'|x'), \tag{2.10}$$

into Eq. (2.8), we arrive at the following set of equations for the coefficients $G_{nn'}$ of that series which we call hereafter the mode Green functions:

$$\left\{\frac{\partial^2}{\partial x^2}+k_F^2+i0-\left(\frac{\pi n}{w(x)}\right)^2[1+\xi_c'^2(x)]-\left(\frac{w'(x)}{2w(x)}\right)^2\right.$$
$$\left.\times\left[1+\frac{(\pi n)^2}{3}\right]\right\}G_{nn'}(x,x')-\frac{4}{w(x)}\sum_{\substack{m=1\\(m\neq n)}}^{\infty}A_{nm}$$
$$\times\left[\hat{\mathcal{U}}_\xi(x)-C_{nm}\frac{\xi_c'(x)w'(x)}{w(x)}\right]G_{mn'}(x,x')$$
$$+\frac{2}{w(x)}\sum_{\substack{m=1\\(m\neq n)}}^{\infty}B_{nm}\left[\hat{\mathcal{U}}_w(x)-C_{nm}\frac{w'^2(x)}{w(x)}\right]G_{mn'}(x,x')$$
$$=\delta_{nn'}\delta(x-x'). \tag{2.11}$$

The numerical coefficients in Eq. (2.11) have the forms

$$A_{nm}=\frac{nm}{n^2-m^2}\sin^2\left[\frac{\pi}{2}(n-m)\right],$$
$$B_{nm}=\frac{nm}{n^2-m^2}\cos^2\left[\frac{\pi}{2}(n-m)\right], \tag{2.12}$$
$$C_{nm}=\frac{3n^2+m^2}{n^2-m^2},$$

$\hat{\mathcal{U}}_{\xi,w}(x)$ are differential operators of the types

$$\hat{\mathcal{U}}_\xi(x)=\xi_c'(x)\frac{\partial}{\partial x}+\frac{\partial}{\partial x}\xi_c'(x), \tag{2.13a}$$

$$\hat{\mathcal{U}}_w(x)=w'(x)\frac{\partial}{\partial x}+\frac{\partial}{\partial x}w'(x). \tag{2.13b}$$

The conductance expression [Eq. (2.7)], on substituting the Green function [Eq. (2.10)] and integrating over a coordinate y, is reduced to the form

$$g(L)=-\frac{4}{L^2}\int_{-L/2}^{L/2}dx\int_{-L/2}^{L/2}dx'\sum_{n,n'=1}^{N_c}\left[\frac{\partial G_{nn'}(x,x')}{\partial x}\right.$$
$$\left.+\sum_{\substack{m=1\\(m\neq n)}}^{\infty}\Phi_{nm}(x)G_{mn'}(x,x')\right]\left[\frac{\partial G^*_{nn'}(x,x')}{\partial x'}\right.$$
$$\left.-\sum_{\substack{m'=1\\(m'\neq n')}}^{\infty}G^*_{nm'}(x,x')\Phi_{m'n'}(x')\right]. \tag{2.14}$$

Here $N_c=[kD/\pi]$ is the number of "open conducting channels," that is, extended waveguide modes. The coefficient matrix $\{\Phi_{nm}\}$ in Eq. (2.14) is composed of the elements

$$\Phi_{nm}(x)=2B_{nm}\frac{w'(x)}{w(x)}(1-\delta_{nm})-4A_{nm}\frac{\xi_c{}'(x)}{w(x)}, \tag{2.15}$$

from which it is evident that, subject to Eq. (2.12), there are no diagonal terms: $\Phi_{nn}(x)\equiv 0$.

We emphasize that the set of mode equations (2.11)–(2.15) is applicable for any correlation model of rough boundaries. Equation (2.11) for the mode Green functions contains the effective electron-surface scattering potential which consists of two (substantially different in their physical meaning) types of terms. The first term $[\pi n/w(x)]^2$ is determined by a random local displacement of the waveguide boundaries and, therefore, is responsible for the BH scattering. All the other terms contain the gradients either of the wire width $w'(x)$ or of the y coordinate of the symmetry line, $\xi_c'(x)$. Consequently, they describe the BS electron scattering. Note that the structure of the BS part of the scattering operator is not highly sensitive to the difference between two specific models of the waveguide, when either $w'(x)\equiv 0$ or $\xi_c'(x)\equiv 0$. At the same time, depending on these models, the role of the BH term in the electron scattering changes drastically. Indeed, if $w'(x)\equiv 0$ [i.e., $w(x)=\text{const}$], the electron scattering arises only due to fluctuations of the roughness slopes that are described by the BS terms with $\xi_c'(x)$. Otherwise, when $\xi_c'(x)\equiv 0$ both the BS and BH scattering potentials are contributing. The case of $w'(x)\equiv 0$ corresponds to the CCB model of the waveguide[11] (constant width, bend-type inhomogeneities) whereas the case $\xi_c'(x)\equiv 0$ is physically equivalent to the SSB model with the correlation properties [Eqs. (2.4)–(2.6)] ("straight" waveguide with fluctuating width). So, we can conclude that the SSB model is qualitatively similar to the general one with arbitrary correlation properties of the lateral boundaries. On the other hand, employing this model allows one to avoid excessively cumbersome calculations without sacrificing the quality of the results.

III. REDUCTION TO ONE-DIMENSIONAL DYNAMIC PROBLEM

In studying the electron transport in bounded systems, one of the basic characteristics is the number of conducting channels or, what is the same, the extended waveguide modes. However, the number of eigenmodes cannot in all cases serve as a good quantum parameter if one addresses a waveguide with a variable cross section. Nevertheless, a formulation of the investigated problem in a mode representation is advantageous, since in this approach the mode structure of the conductor can always be fixed, $N_c=\text{const}$, while a boundary displacement can be considered as a source of perturbation of the mode Hamiltonian in Eq. (2.11).

In this work, bearing in mind the perspective of Anderson localization of the current carriers owing to such "edge perturbations," we consider a *single-mode* strip with the average width D confined within the interval

$$\pi/k_F<D<2\pi/k_F. \tag{3.1}$$

For the reasons being discussed below (also see the Appendix), fluctuations of the conductor width will be regarded to be small as compared to its average value.

Under restrictions (3.1), the only element of the mode Green function matrix $\{G_{nn'}\}$ whose contribution to the conductance [Eq. (2.14)] at weak scattering [see Eq. (4.1)] is not parametrically small is the *intramode* propagator $G_{11}(x,x')$. Before deriving the closed equation for this function, it is worth noting that under certain conditions the operator potentials $\hat{\mathcal{U}}_{\xi,w}(x)$ can be left alone in square brackets standing in front of the *intermode* propagator $G_{mn'}$ in Eq. (2.11). Indeed, the relative part of the terms quadratic in roughness slope and the linear terms in those brackets is estimated by the ratio

$$\frac{\xi'^2(x)}{w(x)|k\xi'(x)+\xi''(x)|}\sim\frac{\sigma}{D(1+kr_c)}. \tag{3.2}$$

It is evident that if the rms height of the boundary displacement is small enough,

$$\sigma/D\ll 1, \tag{3.3}$$

one can neglect the terms that are quadratic in ξ' in comparison with their adjacent linear counterparts. In addition, for the sake of simplicity of the subsequent calculation we will consider the boundary asperities to be also smooth,

$$\sigma/r_c\ll 1, \tag{3.4}$$

yet without requiring their adiabaticity in the sense of Refs. 24 and 25.

Limitations (3.3) and (3.4) are common in solving the problems of classical wave scattering using the perturbation theory.[18] The restrictions are motivated by the necessity of eliminating the well-known ''shadowing'' effect. In view of likeness of the mathematical technique, this issue arises in the considered quantum problem as well. However, under conditions (3.3) and (3.4) there does not exist the wave shadowing in a single-mode waveguide.

With all the above reasonings taken into account, we obtain the equation for the Green function $G_{11}(x,x')$, setting $n=n'=1$ in Eq. (2.11):

$$\left[\frac{\partial^2}{\partial x^2}+k_1^2+i0-V_h(x)-V_s(x)\right]G_{11}(x,x') - \sum_{m=2}^{\infty}\hat{U}_{1m}(x)G_{m1}(x,x')=\delta(x-x'). \tag{3.5}$$

Here we have introduced the notation k_1^2 for the ''unperturbed'' lengthwise energy of the extended mode $n=1$:

$$k_1^2=k_F^2-\left\langle\frac{\pi^2}{w^2(x)}\right\rangle-\left(1+\frac{\pi^2}{3}\right)\left\langle\left[\frac{w'(x)}{2w(x)}\right]^2\right\rangle. \tag{3.6}$$

The quantities $V_h(x)$ and $V_s(x)$ in the square brackets of Eq. (3.5) stand for the *intramode* potentials, and have the forms

$$V_h(x)=\frac{\pi^2}{w^2(x)}-\left\langle\frac{\pi^2}{w^2(x)}\right\rangle, \tag{3.7a}$$

$$V_s(x)=\left(1+\frac{\pi^2}{3}\right)\left\{\left[\frac{w'(x)}{2w(x)}\right]^2-\left\langle\left[\frac{w'(x)}{2w(x)}\right]^2\right\rangle\right\}. \tag{3.7b}$$

Assuming these potentials to be taken as a perturbation, we design them purposefully to have $\langle V_{h,s}(x)\rangle=0$. As to the *intermode* operator potential in Eq. (3.5),

$$\hat{U}_{1m}(x)=-B_{1m}\frac{2}{w(x)}\hat{\mathcal{U}}_w(x), \tag{3.8}$$

it possesses this property by definition.

Although the mode Green functions in Eq. (3.5) depend on a single space variable, the problem certainly cannot be thought of as one dimensional for coupling the function $G_{11}(x,x')$ to all intermode propagators with mode indices $m\neq 1$. Nevertheless, one can obtain a closed equation for this function, which at weak scattering permits a perturbation analysis adequate to the diagrammatic method of Refs. 26 and 16. In Ref. 27, the evidence was given that in the case of an arbitrary 2D imperfect waveguide system governed by a set of dynamic equations of the same functional structure as that of the system (2.11), all of the intermode propagators $G_{mn}(x,x')$ can be expressed, by means of a linear operator, in terms of only one intramode Green function $G_{nn}(x,x')$. Although for arbitrarily disordered systems this relation is of little practical value for the complex dependence on scattering potentials, under weak-scattering conditions the reciprocal operator expressions prove to be rather uncomplicated and readily analyzable. Referring the interested reader to Ref. 27 for the exact procedure, here we give a simple recipe for obtaining the approximate relation between the intermode and intramode propagators, which is valid in the case of weak electron scattering.

This recipe, already used in Ref. 11, consists of solving the set of equations (2.11) iteratively with respect to the intermode propagators entering Eq. (3.5). By letting $n'=1$ and redesignating the mode variables, one can reduce Eq. (2.11) to a set of inhomogeneous equations with respect to the functions $G_{m1}(x,x')$ with $m>1$:

$$\hat{\mathcal{G}}_m^{-1}G_{m1}(x,x')-\sum_{\substack{k=2\\(k\neq m)}}^{\infty}\hat{U}_{mk}(x)G_{k1}(x,x') =\hat{U}_{m1}(x)G_{11}(x,x'). \tag{3.9}$$

It can be easily seen that all interesting propagators G_{m1} can be expressed linearly through the single intramode propagator G_{11}. In Eq. (3.9), $\hat{\mathcal{G}}_m^{-1}$ is the differential operator from curly brackets of Eq. (2.11), where the term with $\xi_c'^2(x)$ should be omitted, in accordance with the SSB model chosen, and replacement of mode indices should be made $n\to m$. The intermode potentials $\hat{U}_{mk}(x)$ in Eq. (3.9) have a form similar to operator (3.8):

$$\hat{U}_{mk}(x)=-B_{mk}\frac{2}{w(x)}\hat{\mathcal{U}}_w(x). \tag{3.10}$$

It is essential that in the case of weak scattering all Green functions $\mathcal{G}_m(x,x')$, with the aid of which Eq. (3.9) is to be solved, belong to the class of so-called *evanescent* functions. They are sufficient to be taken in the unperturbed form (i.e., to zeroth order in boundary roughness) which are found to be strongly localized:

$$\mathcal{G}_m^{(0)}(x,x')=-\frac{1}{2|k_m|}\exp(-|k_m||x-x'|). \tag{3.11}$$

Here $|k_m|=[(\pi m/D)^2-k_F^2]^{1/2}>0$. Since at weak scattering all of the intermode potentials entering Eq. (3.9) can be regarded as small (in functional sense), the following approximate relation can be derived iteratively:

$$G_{m1}(x,x')\approx\int_{-L/2}^{L/2}dx_1\mathcal{G}_m^{(0)}(x,x_1)\hat{U}_{m1}(x_1)G_{11}(x_1,x'). \tag{3.12}$$

By substituting Eq. (3.12) into Eq. (3.5), we eventually arrive at a closed equation for the intramode propagator G_{11}:

$$\left[\frac{\partial^2}{\partial x^2}+k_1^2+i0-V_h(x)-V_s(x)\right]G_{11}(x,x') - \int_{-L/2}^{L/2}dx_1\hat{K}(x,x_1)G_{11}(x_1,x')=\delta(x-x'). \tag{3.13}$$

Equation (3.13) contains complete information on the scattering of single-extended mode electrons by the roughness of the conductor boundaries. While local potentials $V_h(x)$ and $V_s(x)$ are responsible for the intramode scattering, the integral operator in Eq. (3.13) accounts also for the intermode scattering. The kernel of this operator has the form

$$\hat{K}(x,x')=-\sum_{m=2}^{\infty}B_{1m}^2\left[\left(\frac{2}{w(x)}\right)^2\hat{\mathcal{U}}_w(x)\mathcal{G}_m^{(0)}(x,x')\hat{\mathcal{U}}_w(x') - \left\langle\left(\frac{2}{w(x)}\right)^2\hat{\mathcal{U}}_w(x)\mathcal{G}_m^{(0)}(x,x')\hat{\mathcal{U}}_w(x')\right\rangle\right], \tag{3.14}$$

being, in its turn, a differential operator. Similar to the intramode potentials $V_{h,s}(x)$, the operator potential [Eq. (3.14)] is constructed in such a way as to make $\langle\hat{K}(x,x')\rangle=0$. This certainly brings about an even greater complication of the exact form of the longitudinal energy k_1^2 in comparison with that given in Eq. (3.6). But the smoothness condition [Eq. (3.4)] makes it possible to omit the terms containing the derivative $w'(x)$ in the "unperturbed" mode energy, thus replacing it with the simplified value $k_1^2\approx k_F^2-\langle\pi^2/w^2(x)\rangle$.

At last, in formula (2.14) for the conductance, in view of the single-mode geometry of the conducting strip [Eq. (3.1)], one should keep only the terms with the diagonal mode propagator $G_{11}(x,x')$. With Eqs. (2.15) and (3.4), the conductance expression reduces to a relatively simple form

$$g(L)=-\frac{4}{L^2}\int_{-L/2}^{L/2}dx\int_{-L/2}^{L/2}dx'\frac{\partial G_{11}(x,x')}{\partial x}\,\frac{\partial G_{11}^*(x,x')}{\partial x'}, \tag{3.15}$$

which will be the subject of further analysis in conjunction with Eq. (3.13).

IV. TWO-SCALE APPROACH TO THE PROPAGATOR CALCULATION

Since we focus our attention on the single-mode quantum wires, the unperturbed Green functions [Eqs. (3.11)] of all higher modes with $m\geqslant 2$ are real valued and strongly localized in space. The kernel of the operator potential in Eq. (3.13) is thus Hermitian, which ensures the Hermicity of the total Hamiltonian of the dynamic system.

Equation (3.13) can be solved asymptotically within the WS approximation. This approximation is most convenient to state in terms of spatial lengths inherent in the problem. We will arrange these lengths into two groups, viz. "microscopic" and "macroscopic" lengths. The electron Fermi wavelength $2\pi k_F^{-1}$ and the correlation length of boundary asperities, r_c, are assigned to the first group. Among the macroscopic lengths there will be a scattering l_{sc} of the extended mode $n=1$ (to be determined below) and the length L of an irregular part of the conductor. In terms of these lengths, the criterion for scattering to be classified as weak can be expressed through the following inequalities:

$$k_F^{-1},r_c\ll l_{sc},L. \tag{4.1}$$

Note that the correlation between lengths pertaining to the same group (either microscopic or macroscopic) can be thought of as arbitrary.

The retarded Green function entering the conductance expression [Eq. (3.15)] is a solution to the *boundary-value* problem defined by Eq. (3.13) and the appropriate boundary conditions. We assume the conducting strip to be open at the ends $x=\pm L/2$ in the direction of the current. For an open waveguide system, Sommerfeld's radiation conditions well-known in classical wave theory[18,28] are appropriate. In the case of 1D equation (3.13) they can be expressed in Leontovich's form[18,29]

$$\left(\frac{\partial}{\partial x}\mp ik_1\right)G_{11}(x,x')\bigg|_{x=\pm L/2}=0, \tag{4.2}$$

where the source is assumed to be placed inside the waveguide, $x'\in[-L/2,L/2]$.

For lack of dynamic causality, the solution of Sturm-Liuville problem [Eqs. (3.13) and (4.2)] is determined functionally by the potentials in the bulk of the interval $x\in(-L/2,L/2)$. This makes it rather difficult to obtain the correlation functions by applying the well-elaborated methods of statistical analysis which are applicable to the evolution-type problems. One of the commonly used methods aimed at reducing boundary-value problems to evolutionary ones is the invariant imbedding method.[29,30] Yet in this study we apply a different approach which seems to be more general. We will search for Green function of Eq. (3.13) in the form

$$G_{11}(x,x')=\mathcal{W}^{-1}[\psi_+(x)\psi_-(x')\Theta(x-x') + \psi_+(x')\psi_-(x)\Theta(x'-x)], \tag{4.3}$$

where $\psi_{\pm}(x)$ are linearly independent solutions of the homogeneous equation (3.13) supplemented with radiation conditions analogous to Eq. (4.2) at only one of the strip ends, $x=\pm L/2$, in accordance with the "sign" index of $\psi_{\pm}$. The Wronskian of those functions is $\mathcal{W}$, and $\Theta(x)$ is the Heaviside unit-step function.

It is advantageous for the further analysis to represent the functions $\psi_{\pm}(x)$ as superpositions of modulated harmonic waves propagating in opposite directions along the x axis:

$$\psi_{\pm}(x)=\pi_{\pm}(x)\exp(\pm ik_1x)-i\gamma_{\pm}(x)\exp(\mp ik_1x). \tag{4.4}$$

The WS approximation expressed in terms of inequalities (4.1) suggests that the "amplitudes" $\pi_{\pm}(x)$ and $\gamma_{\pm}(x)$ in Eq. (4.4) may be thought of as varying slowly as compared to the "fast" exponentials $\exp(\pm ik_1x)$. This makes it possible to obtain "truncated" equations for those amplitudes by averaging the exact Schrödinger equation over "rapid" phases, as it is done in the theory of nonlinear oscillations.[31] Specifically, we multiply both sides of the homogeneous equation (3.13) from the left by the exponent function $\exp(\mp ik_1x)$, and then average all the terms over space interval $2l$, which can be chosen to have the arbitrary length between the microscopic and macroscopic lengths of the problem:

$$k_F^{-1},r_c\ll l\ll l_{sc},L. \tag{4.5}$$

The final result is not expected to depend exactly on the choice of the averaging interval.

By means of such a subaveraging, we arrive at the following set of first-order differential equations for the smooth amplitudes:

$$\begin{aligned}&\pm\pi'_{\pm}(x)+i\eta(x)\pi_{\pm}(x)+\zeta^*_{\pm}(x)\gamma_{\pm}(x)=0,\\&\pm\gamma'_{\pm}(x)-i\eta(x)\gamma_{\pm}(x)+\zeta_{\pm}(x)\pi_{\pm}(x)=0.\end{aligned} \tag{4.6}$$

The radiation conditions for $\psi_{\pm}(x)$ are reformulated as the following "initial" conditions for the functions $\pi_{\pm}$ and $\gamma_{\pm}$:

$$\pi_{\pm}(\pm L/2)=1,\quad \gamma_{\pm}(\pm L/2)=0. \tag{4.7}$$

V. STATISTICAL PROPERTIES OF SMOOTHED POTENTIALS

At the last stage of developing the averaging procedure we specify statistical properties of the functions $\eta(x)$ and $\zeta_{\pm}(x)$ entering Eqs. (4.6). These random fields are defined as narrow packets of spatial harmonics of the initial potentials from the left-hand side of Eq. (3.13):

$$\eta(x)=\frac{1}{2k_1}\int_{x-l}^{x+l}\frac{dt}{2l}\left[V(t)+\int_{-L/2}^{L/2}dx_1e^{-ik_1t}\hat{K}(t,x_1)e^{ik_1x_1}\right], \tag{5.1}$$

$$\zeta_{\pm}(x)=\frac{1}{2k_1}\int_{x-l}^{x+l}\frac{dt}{2l}\left[e^{\pm 2ik_1t}V(t)+\int_{-L/2}^{L/2}dx_1e^{\pm ik_1t}\hat{K}(t,x_1)\mathrm{e}^{\pm ik_1x_1}\right]. \tag{5.2}$$

Unlike the local potential $V(x)=V_h(x)+V_s(x)$, the operator potential $\hat{\mathcal{K}}$, which is specified by the kernel [Eq. (3.14)], is, strictly speaking, nonlocal (the integrodifferential operator). Therefore, its part in the random fields $\eta(x)$ and $\zeta_{\pm}(x)$ is described by more complicated expressions than those given by $V(x)$. In calculation of different correlation functions the random fields $\eta(x)$ and $\zeta_{\pm}(x)$ are not of importance by themselves, but only their statistical moments. Subject to WS conditions, the latter can be calculated no matter what the locality of the corresponding potential may be.

At weak scattering, all the potentials in Eq. (3.13) may be thought of as Gaussian distributed.[32] Consequently, a knowledge of binary correlators of those potentials is sufficient to govern the statistics of all physical quantities. It was shown in Ref. 11 that under restrictions (4.5) for the averaging interval l, only a pair of correlators of the potentials $\eta(x)$ and $\zeta_{\pm}(x)$ are not parametrically small, viz. $\langle\eta(x)\eta(x')\rangle$ and $\langle\zeta_{\pm}(x)\zeta^*_{\pm}(x')\rangle$. Regardless of the potential being local or not, a calculation of these correlators yields

$$\langle\eta(x)\eta(x')\rangle=\frac{1}{L_f}F_l(x-x'), \tag{5.3a}$$

$$\langle\zeta_{\pm}(x)\zeta^*_{\pm}(x')\rangle=\frac{1}{L_b}F_l(x-x'). \tag{5.3b}$$

The function

$$F_l(x)=\int_{-\infty}^{\infty}\frac{dq}{2\pi}e^{iqx}\frac{\sin^2(ql)}{(ql)^2}=\frac{1}{2l}\left(1-\frac{|x|}{2l}\right)\Theta(2l-|x|) \tag{5.4}$$

in Eqs. (5.3) is sharp on the scale of macroscopic lengths, so hereinafter it can be regarded as the δ function in the "distributional" sense, $F_l(x)\to\delta(x)$.

The coefficients $L_{f,b}^{-1}$ in Eqs. (5.3) represent the inverse scattering lengths, forward (f) and backward (b), respectively. They are contributed by all the potentials that result in both BH and BS scattering in Eq. (3.13). In calculating these coefficients, it should be borne in mind that one can disregard the correlation between the "height" potential [Eq. (3.7a)] and the "slope" potentials [Eqs. (3.7b) and (3.14)] because of the height fluctuation $\xi(x)$ and slope $\xi'(x)$ being uncorrelated in WS limit.[18] This permits us to perceive potential (3.7a), on the one hand, and potentials (3.7b) and (3.14), on the other hand, as being associated with some different additive and mutually noninterfering scattering mechanisms. In such a way we obtain

$$1/L_{f,b}=1/L_{f,b}^{(s)}+1/L_{f,b}^{(h)}. \tag{5.5}$$

Scattering lengths $L_{f,b}^{(s)}$ (both forward and backward) associated with "slope" potentials similar to Eqs. (3.7b) and (3.14) were previously studied in Ref. 11 for a CCB model of a roughly bounded strip. Although the CCB waveguide is somewhat different from the SSB waveguide addressed here, the difference appears to arise only in numerical factors at the potentials considered in Ref. 11 and potentials [Eqs. (3.7b) and (3.14)] of this work. Therefore, referring the reader to Ref. 11 for technical details, here we present the final expressions for the inverse scattering lengths associated with BS scattering, which are valid for the SSB waveguide model:

$$\frac{1}{L_f^{(s)}}=\frac{1}{2k_1^2}\left(\frac{\sigma}{D}\right)^4\int_{-\infty}^{\infty}\frac{dq}{2\pi}q^4\widetilde{\mathcal{W}}^2(q)\times\left\{\left(1+\frac{\pi^2}{3}\right)+2\sum_{m=2}^{\infty}B_{1m}^2[(2k_1+q)^2\widetilde{\mathcal{G}}_m^{(0)}(k_1+q)+(2k_1-q)^2\widetilde{\mathcal{G}}_m^{(0)}(k_1-q)]\right\}^2, \tag{5.6a}$$

$$\frac{1}{L_b^{(s)}}=\frac{1}{2k_1^2}\left(\frac{\sigma}{D}\right)^4\int_{-\infty}^{\infty}\frac{dq}{2\pi}(q^2-k_1^2)^2\widetilde{\mathcal{W}}(q-k_1)\widetilde{\mathcal{W}}(q+k_1)\times\left[\left(1+\frac{\pi^2}{3}\right)+4\sum_{m=2}^{\infty}B_{1m}^2(q^2-k_1^2)\widetilde{\mathcal{G}}_m^{(0)}(q)\right]^2. \tag{5.6b}$$

The functions $\widetilde{\mathcal{W}}(q)$ and $\widetilde{\mathcal{G}}_m^{(0)}(q)$ in Eqs. (5.6) are the Fourier transforms of the correlation function $\mathcal{W}(x)$ from Eq. (2.4) and the evanescent Green function [Eq. (3.11)], respectively.

As regards the BH scattering due to potential (3.7a), the corresponding scattering lengths $L_{f,b}^{(h)}$ is worth considering here in more detail. At first glance, it seems natural to find their by expanding the potential (3.7a), with condition (3.3), in small fluctuations of the conductor width:

$$V_h(x)\approx-\frac{2\pi^2}{D^3}\Delta w(x). \tag{5.7}$$

However, in trying to improve the obtained inverse scattering lengths by retaining the terms of higher order in $\Delta w(x)$ it turns out that the corresponding series converges nonuniformly, so that for its convergence it is necessary to hold a great number of terms. The similar problem was encountered earlier in Ref. 33, where the artificial ''cutting'' parameter was introduced for the corresponding series to become ultimately convergent.

This difficulty can be overcome if the expansion of the BH potential in a series of small displacement of the confining surfaces is discarded, but the exact expression [Eq. (3.7a)] is used instead. This can be easily done for the gaussian roughness model, the appropriate scheme being presented in the Appendix. With this technique, for inverse scattering lengths pertaining to potential (3.7a), one can obtain the expressions below, which are valid in the case of small boundary asperities [Eq. (3.3)]:

$$\frac{1}{L_f^{(h)}}=\frac{4\pi^4\sigma^2}{k_1^2D^6}\widetilde{\mathcal{W}}(0), \tag{5.8a}$$

$$\frac{1}{L_b^{(h)}}=\frac{4\pi^4\sigma^2}{k_1^2D^6}\widetilde{\mathcal{W}}(2k_1). \tag{5.8b}$$

It is noteworthy that extinction lengths (5.8), being obtained by rigorous calculation, coincide exactly in form with those obtained by means of the lowest-order expansion [Eq. (5.7)] of Hamiltonian (3.7a) in powers of $\Delta w(x)$.

VI. CONDUCTANCE AND RESISTIVITY MOMENTS

To perform a statistical averaging based on the effective zero-scale correlation of all random potentials [see Eqs. (5.3) and (4.5)], it is necessary to express the conductance [Eq. (3.15)] in terms of smooth amplitudes $\pi_\pm$ and $\gamma_\pm$. Under WS conditions [Eqs. (4.1)], when substituting the Green function in the form of Eq. (4.3) into expression (3.15), it is sufficient to differentiate over coordinate variables only fast exponentials in wave functions (4.4). As a result, the conductance takes on the intermediate form

$$g(L)=\frac{4k_1^2}{L^2|\mathcal{W}|^2}\int_{-L/2}^{L/2}dx\left\{[|\pi_+(x)|^2-|\gamma_+(x)|^2]\times\int_{-L/2}^{x}dx'[|\pi_-(x')|^2-|\gamma_-(x')|^2]+[|\pi_-(x)|^2-|\gamma_-(x)|^2]\times\int_{x}^{L/2}dx'[|\pi_+(x')|^2-|\gamma_+(x')|^2]\right\}. \tag{6.1}$$

Within the same accuracy, the Wronskian $\mathcal{W}$ in Eq. (6.1) is equal to

$$\mathcal{W}\approx2ik_1[\pi_+(x)\pi_-(x)+\gamma_+(x)\gamma_-(x)]=2ik_1\pi_\pm(\mp L/2), \tag{6.2}$$

where the last equality is a consequence of ''boundary'' conditions (4.7), which are valid within the WS approximation as well.

Formula (6.1) can be simplified if the symmetry properties of Eq. (3.13) and the hermicity of the corresponding 1D Hamiltonian are taken into account. Since the problem of Eqs. (4.6) and (4.7) is of evolutionary type, its solution can be represented in terms of an *x*-ordered matrix exponential:

$$\mathbf{I}_\pm(x)=\hat{T}_x\exp\left[\pm\int_x^{\pm L/2}dx'\mathbf{b}(x')\right]. \tag{6.3}$$

Here the matrices of smooth amplitudes $\mathbf{I}_\pm(x)$ have the forms

$$\mathbf{I}_+(x)=\begin{pmatrix}\pi_+(x) & \gamma_+(x)\\ \gamma_+^*(x) & \pi_+^*(x)\end{pmatrix},\quad \mathbf{I}_-(x)=\begin{pmatrix}\pi_-(x) & \gamma_-^*(x)\\ \gamma_-(x) & \pi_-^*(x)\end{pmatrix}. \tag{6.4}$$

$\mathbf{b}(x)$ is a random field matrix,

$$\mathbf{b}(x)=\begin{pmatrix}i\eta(x) & \zeta_+(x)\\ \zeta_-(x) & -i\eta(x)\end{pmatrix}, \tag{6.5}$$

whose off-diagonal elements are interconnected by the equality $\zeta_-(x)=\zeta_+^*(x)$. The operator $\hat{T}_x$ in Eq. (6.3) arranges the multipliers in each of the terms of the exponential series in order of decreasing their coordinate arguments from left to right.

Taking advantage of the operator identity $\ln\det\mathbf{A}\equiv\mathrm{Sp}\ln\mathbf{A}$, matrices (6.4) can be shown to be unimodular,

$$\det\mathbf{I}_{\pm}(x)=|\pi_{\pm}(x)|^2-|\gamma_{\pm}(x)|^2=1. \tag{6.6}$$

This relation, along with equality (6.2), results in the following form of the conductance of a single-mode wire:

$$g(L)=|\pi_{\pm}^{-1}(\mp L/2)|^2. \tag{6.7}$$

Holding to Landauer's concept, the quantity $\pi_{\pm}^{-1}(\mp L/2)$ is to be interpreted as the transmission coefficient of a single-mode quantum waveguide of length L. This interpretation is supported by the following argumentation. From the structure of wave functions (4.4) it follows that the ratio $\Gamma_{+}(x)=\gamma_{+}(x)/\pi_{+}(x)$ is defined as the reflection coefficient for the harmonics k_1 incident onto the interval $(x,L/2)$ with a unit amplitude from the left-hand side. Correspondingly, $\Gamma_{-}(x)=\gamma_{-}(x)/\pi_{-}(x)$ represents the reflection coefficient of the harmonics $-k_1$ incident onto the interval $(-L/2,x)$ from the right-hand side. Subject to this definition, Eq. (6.6) can be rewritten in the form of the conservation law in a non-dissipative medium,

$$|\Gamma_{\pm}(x)|^2+|\pi_{\pm}^{-1}(x)|^2=1, \tag{6.8}$$

whereupon the interpretation of quantity (6.7) as a square modulus of the transmission coefficient of the disordered interval $(-L/2,L/2)$ seems to be apparent.

With conservation law (6.8), it is convenient to perform subsequent calculations of the statistical moments of the conductance using the equation for $\Gamma_{\pm}(x)$ rather than that for the transmission coefficient $\pi_{\pm}^{-1}(x)$. From Eqs. (4.6), the reflection coefficient can be found to obey the Riccati-type closed equation subject to the zero initial condition:

$$\pm\frac{d\Gamma_{\pm}(x)}{dx}=2i\,\eta(x)\Gamma_{\pm}(x)+\zeta_{\pm}^{*}(x)\Gamma_{\pm}^{2}(x)-\zeta_{\pm}(x), \tag{6.9}$$

$$\Gamma_{\pm}(\pm L/2)=0.$$

The forward-scattering random field $\eta(x)$ may be eliminated from Eqs. (6.9) by concurrent phase transformation of the function $\Gamma_{\pm}(x)$ and the backscattering field $\zeta_{\pm}(x)$:

$$\Gamma_{\pm}(x)=\Gamma_{\pm}^{(new)}(x)\exp\left[\pm 2i\int_{\pm L/2}^{x}dx'\,\eta(x')\right], \tag{6.10a}$$

$$\zeta_{\pm}(x)=\zeta_{\pm}^{(new)}(x)\exp\left[\pm 2i\int_{\pm L/2}^{x}dx'\,\eta(x')\right]. \tag{6.10b}$$

This transformation keeps the conductance [Eq. (6.7)] and the correlation relation (5.3b) unaffected, so one may put function $\eta(x)$ in Eq. (6.9) equal to zero. As a consequence, the outcome for an arbitrary moment of the conductance is to be specified exclusively by the *backscattering* of the electrons, i.e. it will depend on the scattering length which inverse value is the sum of inverse lengths [Eqs. (5.6b) and (5.8b)].

To proceed further, consider the nth moment of the local reflection coefficient squared modulus:

$$R_n^{\pm}(x)=\langle|\Gamma_{\pm}(x)|^{2n}\rangle. \tag{6.11}$$

Since the stochastic problem [Eq. (6.9)] is of evolutionary type, it can be reduced, via the Furutsu-Novikov formalism,[29] to the differential-difference equation for moments [Eq. (6.11)] (also see Ref. 34):

$$\pm\frac{dR_n^{\pm}(x)}{dx}=-\frac{n^2}{L_b}[R_{n+1}^{\pm}(x)-2R_n^{\pm}(x)+R_{n-1}^{\pm}(x)],$$
$$n=0,1,2,\ldots\ . \tag{6.12}$$

Here $L_b^{-1}=L_b^{(s)-1}+L_b^{(h)-1}$. The initial condition on the coordinate x to the solution of Eq. (6.12) is

$$R_n^{\pm}(\pm L/2)=\delta_{n0}. \tag{6.13}$$

As for the dependence of $R_n^{\pm}(x)$ on the discrete variable n, it follows from definition (6.11) that $R_0^{\pm}(x)=1$ and $R_n^{\pm}(x)\to 0$ as $n\to\infty$.

The solution of Eq. (6.12) that matches all the above mentioned conditions can be expressed through the probability function $P_L^{\pm}(u|x)$ and represented, upon due parametrization, in the form

$$R_n^{\pm}(x)=\int_1^{\infty}du\,P_L^{\pm}(u|x)\left(\frac{u-1}{u+1}\right)^n. \tag{6.14}$$

Correspondingly, the statistical moments of the conductance [Eq. (6.7)] are represented by the following integral:

$$\langle g^n(L)\rangle=\langle(1-|\Gamma_{\pm}(\mp L/2)|^2)^n\rangle$$
$$=\int_1^{\infty}du\,P_L^{\pm}(u|\mp L/2)\left(\frac{2}{u+1}\right)^n. \tag{6.15}$$

To obtain the probability density $P_L^{\pm}(u|x)$ one should substitute $R_n^{\pm}(x)$ in form (6.14) into Eq. (6.12), thus obtaining the Fokker-Plank equation

$$\pm L_b\frac{\partial P_L^{\pm}(u|x)}{\partial x}=-\frac{\partial}{\partial u}(u^2-1)\frac{\partial P_L^{\pm}(u|x)}{\partial u}, \tag{6.16}$$

which is supplemented, according to Eq. (6.13), by the initial condition on the coordinate x,

$$P_L^{\pm}(u|\pm L/2)=\delta(u-1-0). \tag{6.17}$$

In addition, normalization of the function $P_L^{\pm}(u|x)$ to unity is ensured by $R_0^{\pm}(x)=1$. This implies that the distribution function is integrable over the variable u, in particular, at $u\to 1$ and $u\to\infty$.

The solution to Eq. (6.16), which meets the above mentioned requirements, can be found by using the Mehler-Fock transformation, and has the conventional form[32]

$$P_L^{\pm}(\cosh\alpha|x) = \frac{1}{\sqrt{8\pi}}\left(\frac{L\mp 2x}{2L_b}\right)^{-3/2}\exp\left(-\frac{L\mp 2x}{8L_b}\right) \times\int_{\alpha}^{\infty}\frac{v\,dv}{(\cosh v-\cosh\alpha)^{1/2}}\exp\left[-\frac{v^2}{4}\left(\frac{L\mp 2x}{2L_b}\right)^{-1}\right], \tag{6.18}$$

where the change of a variable has been made: $u=\cosh\alpha$, $\alpha\geqslant 0$. With this expression, equality (6.15) yields a relatively simple (as well as suitable to analyze) formula for the nth moment of the dimensionless conductance:

$$\langle g^n(L)\rangle = \frac{4}{\sqrt{\pi}}\left(\frac{L_b}{L}\right)^{3/2}\exp\left(-\frac{L}{4L_b}\right)\int_0^{\infty}\frac{z\,dz}{\cosh^{2n-1}z} \times\exp\left(-z^2\frac{L_b}{L}\right)\int_0^z dy\cosh^{2(n-1)}y, \quad n=0,\pm1,\pm2,\ldots\ . \tag{6.19}$$

Result (6.19) completely determines main averaged transport characteristics of a single-mode conducting strip. In particular, although the conductance itself is not a self-averaged quantity, one can, in principle, calculate its self-averaged logarithm using the whole set of statistical moments [Eq. (6.19)]. However, it is much easier to obtain this quantity directly from equations (4.6), omitting cumbersome manipulations with a logarithmic series of terms (6.19). Specifically, by differentiating the quantity $|\pi_{\pm}^{-1}(x)|^2$ over x, we arrive at the equation

$$\pm\frac{d}{dx}\ln|\pi_{\pm}^{-1}(x)|^2 = \zeta_{\pm}^{*}(x)\Gamma_{\pm}(x)+\zeta_{\pm}(x)\Gamma_{\pm}^{*}(x). \tag{6.20}$$

By integrating Eq. (6.20) over the interval $(-L/2,L/2)$, with Eq. (4.7) taken into account, we immediately obtain the logarithm of the conductance on the left-hand side. Before averaging the terms in the right-hand side of Eq. (6.20), we point out that from Eq. (6.9) it follows that function $\Gamma_{\pm}(x)$, being considered as a functional of random fields $\zeta_{\pm}$ and $\zeta_{\pm}^{*}$, depends on the value of these fields exactly within the interval $(x,\pm L/2)$, according to the sign index $(\pm)$. Since at weak scattering all the above-mentioned random fields can be regarded as Gaussian-distributed functional variables, we can apply the Furutsu-Novikov formalism for its averaging.[29] The average of the first term on the right-hand side of Eq. (6.20) can, therefore, be presented in the form

$$\langle\zeta_{\pm}^{*}(x)\Gamma_{\pm}(x)\rangle = \pm\int_x^{\pm L/2}dx'\langle\zeta_{\pm}^{*}(x)\zeta_{\pm}(x')\rangle\left\langle\frac{\partial\Gamma_{\pm}(x)}{\partial\zeta_{\pm}(x')}\right\rangle = \frac{1}{2L_b}\left\langle\frac{\partial\Gamma_{\pm}(x)}{\partial\zeta_{\pm}(x')}\right\rangle\Bigg|_{x'\to x\pm 0}, \tag{6.21}$$

where we have used the effective δ correlation of the fields $\zeta_{\pm}(x)$ and $\zeta_{\pm}^{*}(x)$. The variational derivative in Eq. (6.21) can be readily calculated with the use of Eq. (6.9), and thus it turns out to be equal exactly to unity. Finally, we arrive at the well-known result for 1D disordered systems,

$$\langle\ln g(L)\rangle = -L/L_b, \tag{6.22}$$

signaling the exponential fall of the conductance with a growing length L at the so-called representative (nonresonant) realizations of the random potential.[32]

VII. DISCUSSION OF THE RESULTS

With the general formula (6.19), we write expressions for the average resistance $\langle g^{-1}(L)\rangle$ and the average conductance $\langle g(L)\rangle$. At $n=-1$ the integrals in Eq. (6.19) can be calculated exactly, so the average resistance is equal to

$$\langle g^{-1}(L)\rangle = \frac{1}{2}\left[1+\exp\left(\frac{2L}{L_b}\right)\right]. \tag{7.1}$$

At $n=1$ the integration can be performed asymptotically in the parameter L/L_b, giving rise to the following expression:

$$\langle g(L)\rangle \approx \begin{cases} 1-L/L_b & \text{if}\quad L/L_b\ll 1\\ 2^{-1}\pi^{5/2}(L/L_b)^{-3/2}\exp(-L/4L_b) & \text{if}\quad L/L_b\gg 1. \end{cases} \tag{7.2}$$

Results (6.22)–(7.2) are completely in line with the concepts of the localization theory for one-dimensional disordered systems. Obvious indications of the ballistic electron transport in short wires can be easily seen, $g(L)\approx 1$ at $L\ll L_b$. Also, no signs of diffusive motion of the electrons in long wires are present in any result. Conversely, in long wires, $L\gg L_b$, resistance (7.1) displays an exponential increase with a growing strip length, and the asymptotic [Eq. (7.2)] shows an exponential decrease of the average conductance as the length L exceeds the value of $4L_b$. This behavior is characteristic of conduction electrons undergoing Anderson localization. The inverse of the quadruple backscattering extinction length is equal to the Lyapunov exponent for the electron wave function in the case of dimension unity,[32] so that the quantity $l_{\mathrm{loc}}=4L_b$ is conventionally called the (one-dimensional) localization length.

Both formulas (6.19) and (6.22), and, hence, Eqs. (7.1) and (7.2), are universal in that they are applicable for any one-dimensional degenerate system subject to weak *static* disorder. Only the particular dependence of scattering lengths $L_{f,b}$ and, consequently, the localization length l_{loc} is determined by the physical nature of disorder. In this paper, we have found that if the boundary roughness proves to be the main cause of the disorder in a 2D single-mode conducting strip, the interpretation of a scattering mechanism may be substantially different, depending on the interrelation between by-slope scattering lengths [Eqs. (5.6)] and by-height lengths [Eqs. (5.8)]. To make a correct comparison of the lengths, the roughness statistics needs to be specified. We will make a comparison for two characteristic models consistent with the demand for analyticity of the random functions $\xi_{1,2}(x)$ descriptive of boundaries of the conductor. Specifically, we examine the case of asperities subject to Gaussian (exponential) correlation statistics, $\mathcal{W}(x)=\exp(-x^2/2r_c^2)$, and those described by the Lorentz (power-type) correlation function $\mathcal{W}(x)=[1+(x/r_c)^2]^{-1}$. For the Gaussian roughness, from Eqs. (5.6b) and (5.8b) estimates can be obtained:

$$\frac{1}{L_b^{(h)}}\bigg|_{\text{Gauss}} \sim \left(\frac{\sigma}{D}\right)^2 \frac{r_c}{D^2}\begin{cases}1 & \text{if } r_c/D\ll 1 \quad (\text{i.e., } k_1 r_c\ll 1)\\ \exp(-2k_1^2 r_c^2) & \text{if } r_c/D\gg 1 \quad (\text{i.e., } k_1 r_c\gg 1),\end{cases} \tag{7.3a}$$

$$\frac{1}{L_b^{(s)}}\bigg|_{\text{Gauss}} \sim \left(\frac{\sigma}{r_c}\right)^4 \frac{r_c}{D^2}\begin{cases}1 & \text{if } r_c/D\ll 1 \quad (\text{i.e., } k_1 r_c\ll 1)\\ (k_1 r_c)^4\exp(-k_1^2 r_c^2) & \text{if } r_c/D\gg 1 \quad (\text{i.e., } k_1 r_c\gg 1).\end{cases} \tag{7.3b}$$

If the height correlation is Lorentzian, the estimates change to the following:

$$\frac{1}{L_b^{(h)}}\bigg|_{\text{Lorentz}} \sim \left(\frac{\sigma}{D}\right)^2 \frac{r_c}{D^2}\begin{cases}1 & \text{if } r_c/D\ll 1 \quad (\text{i.e., } k_1 r_c\ll 1)\\ \exp(-2k_1 r_c) & \text{if } r_c/D\gg 1 \quad (\text{i.e., } k_1 r_c\gg 1),\end{cases} \tag{7.4a}$$

$$\frac{1}{L_b^{(s)}}\bigg|_{\text{Lorentz}} \sim \left(\frac{\sigma}{r_c}\right)^4 \frac{r_c}{D^2}\begin{cases}1 & \text{if } r_c/D\ll 1 \quad (\text{i.e., } k_1 r_c\ll 1)\\ (k_1 r_c)^5\exp(-2k_1 r_c) & \text{if } r_c/D\gg 1 \quad (\text{i.e., } k_1 r_c\gg 1).\end{cases} \tag{7.4b}$$

Based on estimates (7.3) and (7.4), the relative intensity of BS and BH scattering can be estimated as follows:

$$\frac{L_b^{(h)}}{L_b^{(s)}}\bigg|_{\text{Gauss}} \sim \left(\frac{\sigma}{D}\right)^2\left(\frac{D}{r_c}\right)^4\begin{cases}1 & \text{if } r_c/D\ll 1 \quad (\text{i.e., } k_1 r_c\ll 1)\\ (r_c/D)^4\exp(k_1^2 r_c^2) & \text{if } r_c/D\gg 1 \quad (\text{i.e., } k_1 r_c\gg 1),\end{cases} \tag{7.5a}$$

$$\frac{L_b^{(h)}}{L_b^{(s)}}\bigg|_{\text{Lorentz}} \sim \left(\frac{\sigma}{D}\right)^2\left(\frac{D}{r_c}\right)^4\begin{cases}1 & \text{if } r_c/D\ll 1 \quad (\text{i.e., } k_1 r_c\ll 1)\\ (r_c/D)^5 & \text{if } r_c/D\gg 1 \quad (\text{i.e., } k_1 r_c\gg 1).\end{cases} \tag{7.5b}$$

From these estimates it can be seen that, in all of the limiting cases considered here, the characteristic ratio $L_b^{(h)}/L_b^{(s)}$ is determined by the product of a small Rayleigh parameter $(\sigma/D)^2$ and some scale parameter, though individual for different roughness correlation, which depends on the relation between the roughness correlation length and the de Broglie wavelength of the electrons. In the case of small-scale asperities, when $k_F r_c\ll 1$ (or, which is the same, $r_c/D\ll 1$), regardless of the correlation model the relative intensity of BH and BS scattering is characterized by the parameter $(\sigma/D)^2(D/r_c)^4=(\sigma/r_c)^4/(\sigma/D)^2$, which varies over a rather wide range. This is because BH and BS scattering is associated with independent physical origins. Whereas scattering from the potential $V_h(x)$ is governed by the height of the boundary roughness and thus is estimated mostly in terms of the parameter σ/D [Eqs. (7.3a) and (7.4a)], the BS scattering is mostly determined by the slope of the asperities, i.e., by gradients $\xi'_{1,2}(x)$, and is, consequently, governed by the parameter σ/r_c [Eqs. (7.3b) and (7.4b)].

It would be tiresome to discuss here in detail the interrelation between BH and BS scattering mechanisms, assuming the asperities to be large scale, when the parameter $k_F r_c \gg 1$ (i.e., $r_c/D\gg 1$), since in this case the the result depends largely on the correlation model. We leave this particular analysis to an interested reader.

It is noteworthy that in the event when the "slope" mechanism dominates the "height" mechanism, the "surface" scattering rate is proportional to the fourth power of the rms height σ rather than to σ^2, which is customary in diffraction theory.[18] This fact must be taken into account when analyzing experiments aimed at reproducing the surface shape using the data on quantum, as well as classical, wave scattering in rough-bounded waveguide systems.

A remarkable peculiarity of the BS scattering mechanism is that through this scattering the evanescent modes manifest unexpectedly their significance, though they are strongly localized in the direction of propagation and normally do not contribute directly to the energy transport in waveguide systems. In the problem considered here, the evanescent modes are present in the last term on the left-hand side of Eq. (3.13), specifically in the kernel [Eq. (3.14)] of the operator potential $\hat{\mathcal{K}}$. This potential can be seen to govern the intramode scattering of the only propagating mode with $n=1$ through intermode transitions via the virtual evanescent modes with $n\geq 2$. Those transitions contribute to expressions (5.6) of the scattering lengths as much, in order of magnitude, as the direct BS scattering governed by the potential $V_s(x)$ in Eq. (3.13). It should be emphasized that the evanescent modes' contribution to the kinetic coefficients of a mode that is in itself extended seems to be a distinctive feature of wave scattering in surface-corrugated systems. This was proven not to be the case under WS conditions for waveguidelike systems whose inhomogeneity is of the bulk nature.[27]

Apart from the identification of dominant scattering mechanism, specifying scattering lengths [Eqs. (7.3) and (7.4)] for different statistical models of boundary roughness also allows for the criteria of validity of the obtained results in terms of essential physical parameters of the disordered systems. Along with the presumption of smallness and smoothness of boundary asperities (see Eqs. (3.3) and (3.4)) the criteria are dictated by Eq. (4.1) of weak scattering. In specifying those criteria, the smallest of the extinction lengths $L_f^{(h,s)}$ should be taken to substitute for the length l_{sc} in Eq. (4.1) since the inequality $L_f \lesssim L_b$ always holds true.

In conclusion, we make some remarks concerning the methodological side of the problem of wave scattering from rough waveguide surfaces. As far as we know, until the present time there has not been made any reasonable distinction between BH and BS scattering in such systems. Only the existence, in general, of different competing mechanisms responsible for wave scattering from rough surfaces was indicated in Ref. 23 on the basis of the experimental results. Accordingly, the relative function of these scattering mechanisms in dynamic processes in waveguidelike systems was not properly analyzed. Meanwhile, in the course of this work we have made certain that the application of linearized (impedance-type) boundary conditions to a single-mode waveguide is equivalent to retaining in Eq. (3.13) the approximate potential [Eq. (5.7)] instead of its exact value [Eq. (3.7a)], and also disregarding all ''slope'' potentials. However, omitting the latter potentials implies a neglect of the BS scattering mechanism, that is proven to be not always justifiable. The alternative small-slope approximation of Refs. 21 and 22, being guided solely by a slope parameter of the rough surface, does not allow one to separate BH and BS scattering mechanisms as well.

In this work, a method has been suggested, within the framework of which both the above-mentioned scattering mechanisms appear quite naturally to be associated with different terms of the Hamiltonian. We have demonstrated that at least in a single-mode waveguide the scattering caused even by mildly sloping boundary asperities can be attributed to either ''height'' or ''slope'' scattering mechanisms, depending on the statistical properties of the roughness. The competition between these mechanisms is governed by physically different parameters. It is noteworthy that taking into account the BS mechanism is particularly essential if boundary asperities are classified as being large-scale.

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APPENDIX: CORRELATION CHARACTERISTICS OF BY-HEIGHT SCATTERING

The effective subaveraged potentials $\eta_h(x)$ and $\zeta_{h\pm}(x)$ corresponding to the original potential [Eq. (3.7a)] have the forms

$$\eta_h(x)=\frac{\pi^2}{2k_1}\int_{x-l}^{x+l}\frac{dt}{2l}\left[\frac{1}{w^2(t)}-\left\langle\frac{1}{w^2(t)}\right\rangle\right], \qquad \text{(A1a)}$$

$$\zeta_{h\pm}(x)=\frac{\pi^2}{2k_1}\int_{x-l}^{x+l}\frac{dt}{2l}e^{\mp 2ik_1t}\left[\frac{1}{w^2(t)}-\left\langle\frac{1}{w^2(t)}\right\rangle\right]. \qquad \text{(A1b)}$$

To calculate the averages in Eqs. (A1), it is convenient to make use of the methods

$$\left\langle\frac{1}{w^2(t)}\right\rangle=-\frac{\partial}{\partial D}\left\langle\frac{1}{w(t)}\right\rangle, \qquad \text{(A2)}$$

$$\left\langle\frac{1}{w(t)}\right\rangle=\left\langle\frac{1}{D+\Delta\xi(t)}\right\rangle = -i\int_0^\infty dq\langle\exp\{iq[D+\Delta\xi(t)+i0]\}\rangle. \qquad \text{(A3)}$$

In the case of boundary roughness obeying Gaussian statistics, the integral in Eq. (A3) can be averaged without difficulty. Since for the SSB waveguide the equality

$$\langle\Delta\xi(x)\Delta\xi(x')\rangle=4\sigma^2\mathcal{W}(x-x') \qquad \text{(A4)}$$

holds true, the correlator (A3) is equal to

$$\left\langle\frac{1}{w(t)}\right\rangle=-\frac{i}{2\sigma}\sqrt{\frac{\pi}{2}}\exp\left(-\frac{D^2}{8\sigma^2}\right)\operatorname{erfc}\left(\frac{-iD}{\sigma\sqrt{8}}\right), \qquad \text{(A5)}$$

where $\operatorname{erfc}(\cdots)$ is the probability integral (see, e.g., Ref. 35).

With the use of Eq. (A5), the correlator (A2) can be accurately computed. However, a subsequent analysis can be performed analytically only in the case of small-height roughness. With inequality (3.3), averages (A3) and (A2) are asymptotically equal to

$$\left\langle\frac{1}{w(t)}\right\rangle\approx\frac{1}{D}\left[1+4\left(\frac{\sigma}{D}\right)^2\right], \qquad \text{(A6)}$$

$$\left\langle\frac{1}{w^2(t)}\right\rangle\approx\frac{1}{D^2}\left[1+12\left(\frac{\sigma}{D}\right)^2\right]. \qquad \text{(A7)}$$

The next step is a calculation of binary correlation functions $\langle\eta_h(x_1)\eta_h(x_2)\rangle$ and $\langle\zeta_{h\pm}(x_1)\zeta^*_{h\pm}(x_2)\rangle$. In doing so, one has to calculate the correlator

$$\mathcal{L}(t_1-t_2)=\left\langle\left[\frac{1}{w^2(t_1)}-\left\langle\frac{1}{w^2(t_1)}\right\rangle\right]\left[\frac{1}{w^2(t_2)}-\left\langle\frac{1}{w^2(t_2)}\right\rangle\right]\right\rangle$$
$$=\left\langle\frac{1}{w^2(t_1)}\frac{1}{w^2(t_2)}\right\rangle-\left\langle\frac{1}{w^2(t_1)}\right\rangle\left\langle\frac{1}{w^2(t_2)}\right\rangle. \quad \text{(A8)}$$

In the case of small-amplitude roughness, the second term on the right-hand side of Eq. (A8) is governed by asymptotic (A7). As for the first term, for its calculation the method which was already applied to calculate average (A2) is helpful. To this end, we represent the desired correlator in the form

$$\left\langle\frac{1}{w^2(t_1)}\frac{1}{w^2(t_2)}\right\rangle=\lim_{D'\to D}\frac{\partial^2}{\partial D\,\partial D'}\int\int_0^\infty dq_1dq_2 \times\langle\exp\{iq_1[D+\Delta\xi(t_1)+i0] -iq_2[D'+\Delta\xi(t_2)-i0]\}\rangle. \quad \text{(A9)}$$

An averaging of the exponent function in Eq. (A9) readily yields

$$\langle\exp[iq_1\Delta\xi(t_1)-iq_2\Delta\xi(t_2)]\rangle =\exp\{-2(q_1^2+q_2^2)\sigma^2+4q_1q_2\sigma^2\mathcal{W}(t_1-t_2)\}, \quad \text{(A10)}$$

whereupon we arrive, after some tedious manipulations, at the integral representation

$$\left\langle\frac{1}{w^2(t_1)}\frac{1}{w^2(t_2)}\right\rangle =\frac{1}{(2\sigma)^4\sqrt{W_+W_-}}\int_0^\infty du\cos\left(\frac{D}{\sigma\sqrt{2W_+}}u\right)\int_{\varphi(u)}^\infty dv \times\left(\frac{v^2}{W_-}-\frac{u^2}{W_+}\right)\exp\left(-\frac{u^2+v^2}{2}\right). \quad \text{(A11)}$$

Here, for the sake of the formula compactification, we use the notations

$$\varphi(u)=u\sqrt{W_-/W_+},\quad W_\pm=1\pm\mathcal{W}(t_1-t_2). \quad \text{(A12)}$$

Correlation function (A11) can be calculated numerically for the arbitrary value of σ/D. But in the case of small-height roughness the integration in Eq. (A11) can be performed analytically, yielding the asymptotic result

$$\left\langle\frac{1}{w^2(t_1)}\frac{1}{w^2(t_2)}\right\rangle\approx\frac{1}{D^4}\left[1+\left(\frac{2\sigma}{D}\right)^2(5W_++W_-)\right]. \quad \text{(A13)}$$

By substituting Eqs. (A13) and (A7) into Eq. (A8), we obtain

$$\mathcal{L}(t_1-t_2)\approx\frac{16\sigma^2}{D^6}\mathcal{W}(t_1-t_2). \quad \text{(A14)}$$

After a simple integration of correlator (A14), resulting from definition (A1), we arrive at the final expressions for the required correlators:

$$\langle\eta_h(x_1)\eta_h(x_2)\rangle=\frac{1}{L_f^{(h)}}F_l(x_1-x_2), \quad \text{(A15a)}$$

$$\langle\zeta_{h\pm}(x_1)\zeta_{h\pm}^*(x_2)\rangle=\frac{1}{L_b^{(h)}}F_l(x_1-x_2). \quad \text{(A15b)}$$

In Eqs. (A15), the extinction lengths $L_{f,b}^{(h)}$ associated with the height potential [Eq. (3.7a)] are given by Eqs. (5.8).

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[1] N. Trivedi and N. W. Ashcroft, Phys. Rev. B **38**, 12 298 (1988).
[2] N. M. Makarov and I. V. Yurkevich, Zh. Éksp. Teor. Fiz. **96**, 1106 (1989) [Sov. Phys. JETP **69**, 628 (1989)].
[3] V. D. Freilikher, N. M. Makarov, and I. V. Yurkevich, Phys. Rev. B **41**, 8033 (1990).
[4] J. Kierul and J. Ledzion, Phys. Status Solidi A **128**, 117 (1991).
[5] Y. Takagaki and D. K. Ferry, J. Phys.: Condens. Matter **4**, 10 421 (1992).
[6] Chr. Kunze, Solid State Commun. **87**, 359 (1993).
[7] V. I. Kozub and A. A. Krokhin, J. Phys.: Condens. Matter **5**, 9135 (1993).
[8] A. E. Meyerovich and S. Stepaniants, Phys. Rev. B **51**, 17 116 (1995).
[9] N. M. Makarov, A. V. Moroz, and V. A. Yampol'skii, Phys. Rev. B **52**, 6087 (1995).
[10] A. M. Bratkovsky and S. N. Rashkeev, Phys. Rev. B **53**, 13 074 (1996).
[11] N. M. Makarov and Yu. V. Tarasov, J. Phys.: Condens. Matter **10**, 1523 (1998).
[12] J. A. Sánchez-Gil, V. Freilikher, I. V. Yurkevich, and A. A. Maradudin, Phys. Rev. Lett. **80**, 948 (1998).
[13] J. A. Sánchez-Gil, V. Freilikher, A. A. Maradudin, and I. V. Yurkevich, Phys. Rev. B **59**, 5915 (1999).
[14] J. A. Konrady, J. Acoust. Soc. Am. **56**, 1687 (1974).
[15] R. Kubo, J. Phys. Soc. Jpn. **12**, 570 (1957).
[16] A. A. Abrikosov and I. A. Ryzhkin, Adv. Phys. **27**, 147 (1978).
[17] K. B. Efetov, Adv. Phys. **32**, 53 (1983).
[18] F. G. Bass and I. M. Fuks, *Wave Scattering from Statistically Rough Surfaces* (Pergamon, New York, 1979).
[19] A. R. McGurn and A. A. Maradudin, Phys. Rev. B **30**, 3136 (1984).
[20] V. Freilikher and I. Yurkevich, Phys. Lett. A **183**, 247 (1993).

[21] A. G. Voronovich, *Wave Scattering from Rough Surfaces* (Springer Series on Wave Phenomena) (Springer, Berlin, 1994).
[22] A. G. Voronovich, Waves Random Media **6**, 151 (1996).
[23] W. T. Dietze and R. B. Darling, Phys. Rev. B **53**, 3925 (1996).
[24] L. I. Glazman, G. B. Lesovik, D. E. Khmelnitskii, and R. I. Shekhter, Pis'ma Zh. Éksp. Teor. Fiz. **48**, 218 (1988) [JETP Lett. **48**, 238 (1988)].
[25] L. I. Glazman and M. Jonson, Phys. Rev. B **44**, 3810 (1991).
[26] V. L. Berezinski, Zh. Éksp. Teor. Fiz. **65**, 1251 (1973) [Sov. Phys. JETP **38**, 620 (1974)].
[27] Yu. V. Tarasov, Waves Random Media **10**, 395 (2000).
[28] V. S. Vladimirov, *Equations of Mathematical Physics* (Nauka, Moscow, 1967) (in Russian).
[29] V. I. Klyatskin, *The Invariant Imbedding Method in a Theory of Wave Propagation* (Nauka, Moscow, 1986) (in Russian).
[30] R. Bellman and G. M. Wing, *An Introduction to Invariant Imbedding* (Wiley, New York, 1975).
[31] N. N. Bogolyubov and Yu. A. Mitropolsky, *Asymptotic Methods in the Theory of Non-Linear Oscillations* (Nauka, Moscow, 1974) (in Russian).
[32] I. M. Lifshits, S. A. Gredeskul, and L. A. Pastur, *Introduction to the Theory of Disordered Systems* (Wiley, New York, 1988).
[33] A. Kawabata, J. Phys. Soc. Jpn. **82**, 3988 (1993).
[34] M. Ash, W. Kohler, G. Papanicolaou, M. Postel, and B. White, SIAM Rev. **33**, 519 (1991).
[35] *Handbook of Mathematical Functions*, edited by M. Abramowitz and I. A. Stegun (Dover Publications, Inc., New York, 1965).