



# Surface scattering frequency and optical absorptivity of exciton in quasi-two-dimensional quantum wells

N. Atenco-Analco, N.M. Makarov<sup>1</sup>, F. Pérez-Rodríguez\*

*Instituto de Física, Universidad Autónoma de Puebla, Apartado Postal J-48, Puebla 72570, Mexico*

Received 20 December 2000; accepted 8 May 2001 by S.G. Louie

## Abstract

Applying the self-consistent Green's function method, we calculate and analyze the relaxation frequency  $\nu$  of exciton in a quasi-two-dimensional quantum well of arbitrary depth. The exciton relaxation results from the electron and hole scattering by the randomly rough well interfaces. It is shown that the introduced here  $\nu$  controls the behavior of the exciton absorptivity spectrum  $A(\omega)$ . We analyze the line-shape of the ground-state exciton resonance and reveal the criteria for the transition from asymmetric (sharp) to symmetric (broad) resonance. The dependencies of  $\nu$  and  $A$  on the parameters of the exciton quantum well and its rough interfaces are found. © 2001 Elsevier Science Ltd. All rights reserved.

PACS: 71.35.-y; 42.25.Dd; 78.66.-w

Keywords: A. Quantum wells; A. Disordered systems; D. Optical properties

## 1. Introduction

During the past few years, the effect of interface roughness on optical properties of excitons in quantum well structures has been the focus of attention from both experimental and theoretical standpoints (see, e.g. [1–16] and references therein). The theoretical description of exciton–surface interaction is a rather complex problem due to the electron–hole Coulomb attraction, leading to a finite exciton size (Bohr radius)  $a_0$ . So, even near a flat surface, exciton dynamics drastically changes at distances of the order of  $a_0$  [17]. For this reason, optical properties of excitons near rough surfaces of crystals, dominated by bulk behavior, has been mostly studied by employing different phenomenological models [18–24]. Similarly, exciton–surface scattering in quantum wells has been described by using either phenomenological [10,11] or other more sophisticated theories [4–7,13]. The latter are based on the Schrödinger equation for the exciton in-plane center-of-mass motion in a disordered potential. A physically clear analytical solution

of the center-of-mass motion problem can be found within the adiabatic regime [7,25], when  $a_0$  is much less than the correlation length  $R_c$  of the interface roughness. The application of such an approach beyond the adiabatic regime results in an extreme complication of the problem so that it can be solved only numerically [5,6].

Below we apply an approach based on the analysis of the *individual interactions* of the electron and hole with the surface instead of the scattering of their in-plane center-of-mass. The concept of surface-disordered quasi-2D quantum well requires the exciton radius  $a_0$  to be much larger than the average well width  $d$  which should far exceed the r.m.s. roughness height  $\zeta$

$$\zeta \ll d \ll a_0. \quad (1)$$

Due to the right condition, the Coulomb attraction is suppressed in the direction perpendicular to the well plane. Therefore, the interaction of the electron and hole with the rough surface occurs in the presence of their Coulomb in-plane coupling. According to this, the exciton–surface scattering problem can be solved by employing and generalizing to the case of two-particle motion, the self-consistent Green's function method [8] without any a priori restrictions on the roughness correlation length  $R_c$ . This method turns out to be the most appropriate for our purposes

\* Corresponding author. Tel.: +52-22-45-7645; fax: +52-22-44-8947.

E-mail address: fperez@acuaro.ifuap.buap.mx  
(F. Pérez-Rodríguez).

<sup>1</sup> Present address: CIDS, Instituto de Ciencias, UAP.

because it deals with the original exciton Hamiltonian and introduces in a natural way such fundamental microscopic characteristic of exciton as its *surface scattering frequency*  $\nu$ . Besides, the self-consistent approach allows to take into account the inherent action of  $\nu$  on itself and, consequently, to analyze the complete dynamics of the exciton resonance line-shape. The main result of our theory is a relatively simple equation for  $\nu$ , which is valid and easily analyzable for arbitrary relation between the exciton radius  $a_0$  and the roughness correlation length  $R_c$ . It displays explicitly dependencies of  $\nu$  on the parameters of the exciton, interface roughness, and exciton-resonance detuning. We have also studied the correlation between the frequency dependence of  $\nu$  and the line-shape of the absorptivity spectrum for a quasi-2D quantum well.

## 2. Problem formulation

We consider a quantum well occupying the region  $\xi(\vec{r}) \leq z \leq d$ , where  $z$  is the coordinate along the growth direction of the well,  $\xi(\vec{r})$  is a random function of the in-plane position vector  $\vec{r}$  characterized by

$$\langle \xi(\vec{r}) \rangle = 0, \quad \langle \xi(\vec{r}) \xi(\vec{r}') \rangle = \zeta^2 \mathcal{W}(|\vec{r} - \vec{r}'|). \quad (2)$$

The angular brackets stand for an average over the ensemble of realizations of function  $\xi(\vec{r})$ . The correlator  $\mathcal{W}(|\vec{r}|)$  has unit amplitude,  $\mathcal{W}(0) = 1$ , and a typical scale  $R_c$  of monotonous decrease. The chosen system describes two types of surface-disordered quantum wells: (i) one boundary is rough while the other is flat; (ii) both boundaries are rough, statistically identical, and not intercorrelated [26].

Due to the definition (1) of a quasi-2D quantum well, the exciton Hamiltonian  $\hat{\mathcal{H}}_{\text{Q2D}}$  can be written as

$$\hat{\mathcal{H}}_{\text{Q2D}} = E_{\text{gap}} - \frac{\hbar^2}{2M} \frac{\partial^2}{\partial \vec{R}^2} - \frac{\hbar^2}{2\mu} \frac{\partial^2}{\partial \vec{\rho}^2} - \frac{e^2}{\varepsilon_0 \rho} - \frac{\hbar^2}{2m_e} \frac{\partial^2}{\partial z_e^2} - \frac{\hbar^2}{2m_h} \frac{\partial^2}{\partial z_h^2} + U_e(z_e, \vec{r}_e) + U_h(z_h, \vec{r}_h) - i\hbar\nu_0. \quad (3)$$

Here  $E_{\text{gap}}$  is the energy gap between the conduction and valence bands,  $\varepsilon_0$  the dielectric constant of the excitonic medium,  $m_e$  ( $m_h$ ) is the electron (hole) effective mass. The exciton center-of-mass is described by the total mass  $M = m_e + m_h$  and the in-plane radius vector  $\vec{R}$ . The relative electron-hole motion is specified by the reduced mass  $\mu = m_e m_h / M$  and the in-plane vector  $\vec{\rho}$  ( $\vec{r}_{e,h} = \vec{R} \pm \mu \vec{\rho} / m_{e,h}$ ). The quantities  $U_e(z_e, \vec{r}_e)$  and  $U_h(z_h, \vec{r}_h)$  are the confining potentials for the electron and the hole. In the Hamiltonian (Eq. (3)), we have introduced a homogeneous exciton-bulk damping  $\nu_0$  to take into account its effect on the exciton-surface scattering.

The Dyson-type integral equation for the retarded Green's function  $\mathcal{G}$  is obtained by making use of Green's theorem. This equation relates  $\mathcal{G}$  to the Green's function  $\mathcal{G}_0$

for the ideal well with  $\xi(\vec{r}) \equiv 0$  and contains the sum of the *electron-surface*  $\hat{V}_e$  and *hole-surface*  $\hat{V}_h$  scattering operators,

$$\begin{aligned} \hat{V}_{e,h}(\vec{R}, \vec{\rho}, z_{e,h}) &= U_{e,h} [\Theta(\xi(\vec{r}_{e,h}) - z_{e,h}) - \Theta(-z_{e,h})] \\ &= U_{e,h} \delta(z_{e,h}) \xi(\vec{R} \pm \mu \vec{\rho} / m_{e,h}), \end{aligned} \quad (4)$$

where  $\Theta(x)$  is the Heaviside unit-step function and  $\delta(x)$  the Dirac delta-function,  $U_{e,h}$  the height of the potential barrier for the electron (e) and hole (h). We have averaged the equation for  $\mathcal{G}$  by applying the technique proposed in Ref. [27]. The resulting integral equation for the averaged Green's function  $\bar{\mathcal{G}}$  within the self-consistent Born approximation can be symbolically written as

$$\bar{\mathcal{G}} = \mathcal{G}_0 + \mathcal{G}_0 \langle (\hat{V}_e + \hat{V}_h) \bar{\mathcal{G}} (\hat{V}_e + \hat{V}_h) \rangle \bar{\mathcal{G}}. \quad (5)$$

## 3. Results and discussion

At weak surface scattering and with an arbitrary character (adiabatic or nonadiabatic) of the perturbation potentials  $\hat{V}_{e,h}$ , the solution of Eq. (5) can be written as

$$\begin{aligned} \bar{\mathcal{G}}(|\vec{R} - \vec{R}'|; \vec{\rho}, \vec{\rho}'; z_e, z_e'; z_h, z_h') \\ = \sum_{\lambda} \Phi_{nm}(\vec{\rho}) \Phi_{nm}^*(\vec{\rho}') \Psi_{n_e}^{(e)}(z_e) \Psi_{n_e}^{(e)}(z_e') \Psi_{n_h}^{(h)}(z_h) \Psi_{n_h}^{(h)}(z_h') \\ \times \int_{-\infty}^{\infty} \frac{d\vec{k}_t}{(2\pi)^2} \frac{\exp[i\vec{k}_t(\vec{R} - \vec{R}')] }{\hbar[\omega - \omega_{\lambda} - (\hbar k_t^2 / 2M) + i\nu_0 + i\nu_{\lambda}(k_t)]}. \end{aligned} \quad (6)$$

Here  $\lambda = \{n, m; n_e, n_h\}$  is the complete set of the exciton quantum numbers,  $\Phi_{nm}(\vec{\rho})$  are the eigenfunctions for the relative electron-hole motion in the 2D Coulomb potential,  $\Psi_{n_e}^{(e)}(z_e)$  and  $\Psi_{n_h}^{(h)}(z_h)$  are the confinement wave functions for the individual motion of electron and hole,  $\hbar\omega$  and  $\hbar\omega_{\lambda}$  are, respectively, the energies of the exciton and its eigenstate. The quantity  $\hbar\nu_{\lambda}(k_t)$  enters Eq. (6) as the imaginary part of the self-energy and, therefore,  $\nu_{\lambda}(k_t)$  symbolizes the surface scattering frequency of the exciton  $\lambda$ -state.<sup>2</sup> Because of the self-consistency  $\nu_{\lambda}(k_t)$  satisfies an integral equation that directly follows from Eq. (5) and contains a sum over the set  $\lambda'$  of extended modes. We restrict our further consideration to the most important and widely analyzed case of the exciton ground resonance ( $\lambda = \lambda' = \{0, 0; 1, 1\}$ ) and normal incidence of light ( $k_t = 0$ ). In this case, the equation for  $\nu \equiv \nu_{\lambda}(k_t = 0)$  takes a simpler form

$$\frac{\nu}{\nu_l} = \frac{1}{\pi} \int_0^{\infty} d\omega_t \Delta_{\nu_0 + \nu}(\omega_t) Q(\omega_t) \frac{W(\sqrt{2M\omega_t/\hbar})}{W(0)}. \quad (7)$$

<sup>2</sup> The real part of the self-energy gives rise to a roughness-induced shift of the exciton resonance frequency, which in our model is included in  $\omega_{\lambda}$ .

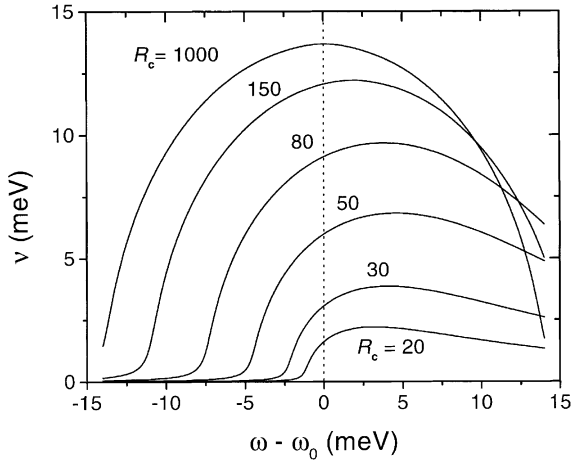


Fig. 1. The exciton–surface scattering frequency  $\nu$  vs. the resonance detuning  $\omega - \omega_0$  for deep GaAs quantum wells with different correlation lengths  $R_c$  (Å). Other parameters used are [31,32]:  $d = 60$  Å,  $\omega_0 = 1.62$  eV corresponds to the ground-state light-hole exciton,  $m_h = 0.21m$ ,  $m_e = 0.067m$  ( $m$  is the free electron mass),  $\nu_0 = 0.1$  meV,  $\zeta = 2$  Å.

The integrand in Eq. (7) consists of three functions. The sharp resonant function

$$\Delta_{\nu_0+\nu}(\omega_t) = \frac{\nu_0 + \nu}{(\omega - \omega_0 - \omega_t)^2 + (\nu_0 + \nu)^2}, \quad (8)$$

having a variation scale  $\nu_0 + \nu$ . The exciton function

$$Q(\omega_t) = \left\{ \frac{\mu}{m_e} b_c \left[ 1 + \frac{2M}{\hbar} \left( \frac{\mu}{m_e} \frac{a_0}{4} \right)^2 \omega_t \right]^{-3/2} + \frac{\mu}{m_h} b_h \left[ 1 + \frac{2M}{\hbar} \left( \frac{\mu}{m_h} \frac{a_0}{4} \right)^2 \omega_t \right]^{-3/2} \right\}^2 \quad (9)$$

has its largest value  $Q(0)$  and decreases with  $\omega_t$  over the scale  $\omega_Q = (\hbar/2M)(\min\{m_e, m_h\}/\mu)^2(4/a_0)^2$ . The quantity  $b_{e,h} = U_{e,h} \Psi_1^{(e,h)2}(0)/\lim_{u_{e,h} \rightarrow \infty} U_{e,h} \Psi_1^{(e,h)2}(0)$  characterizes the finiteness of the potential barrier,  $a_0 = \hbar^2 \epsilon_0 / e^2 \mu$ . At last, the Fourier transform  $W(k_r)$  of the correlator  $\mathcal{W}(|\vec{r}|)$ , as a function of  $\omega_t$ , has the scale of decrease  $\omega_W = \hbar R_c^{-2} / 2M$ . The normalization constant  $\nu_l$  is defined by

$$\nu_l = B \frac{\hbar(\pi d)^2}{2\mu} \frac{M}{\mu} \left( \frac{\zeta}{d} \right)^2 \left( \frac{\pi R_c}{d} \right)^2. \quad (10)$$

Here the positive number  $B = W(0)/R_c^2$  depends neither on  $\zeta$  nor on  $R_c$  and has a value of the order of 1.

Note that only the function (8) in Eq. (7) depends on the resonance detuning  $\omega - \omega_0$ . Therefore, the line-shape of  $\nu(\omega)$  is mainly determined by the relation between the variation scales  $\nu_0 + \nu$  and  $\min\{\omega_W, \omega_Q\}$  of functions  $\Delta$  and  $QW$ , respectively. At the same time, the ratio of  $R_c$  to  $a_0$  and, hence,  $\omega_W$  to  $\omega_Q$  can be arbitrary. Within the

adiabatic regime  $\omega_W \ll \omega_Q$  and, therefore, the exciton function  $Q(\omega_t)$  in Eq. (7) can be replaced by its value  $Q(0)$ . This replacement is equivalent to the neglect of the dependence on the vector  $\vec{\rho}$  ( $\rho \sim a_0$ ) in the scattering potential (4).

In Fig. 1, we present the dependence of  $\nu$  on the resonance detuning  $\omega - \omega_0$  for deep ( $b_{e,h} = 1$ ) GaAs quantum wells with distinct mean lengths  $R_c$  of surface defects (peaks and valleys) and Gaussian correlator  $\mathcal{W}(|\vec{r}|) = \exp(-r^2/R_c^2)$ . For  $R_c = 20$  Å, the ratio  $\omega_W/\omega_Q > 1$ , whereas for the other values of  $R_c$ ,  $\omega_W/\omega_Q < 1$ . The two lower curves correspond to the case of a sharp exciton resonance, when  $\nu_0 + \nu \ll \min\{\omega_W, \omega_Q\}$ . They have the same line-shape, but different value  $\nu_l$ . Here, the surface scattering frequency  $\nu$  increases with  $\omega$  and takes a value  $\nu \sim \nu_l/2$  at the resonance point  $\omega = \omega_0$ . On the right side of the resonance,  $\omega > \omega_0$ , the quantity  $\nu$  reaches its maximum  $\sim \nu_l$  and, afterwards, monotonously decreases. Other curves show the evolution of the frequency dependence of  $\nu$  as  $\omega_W$  is decreased. It is seen that exciton–surface scattering frequency  $\nu$  is enhanced and its maximum is shifted first to the right and later to the left, towards the point  $\omega = \omega_0$ . For large values of  $R_c$  so that  $\nu_0 + \nu \gg \omega_W$  (broad resonance), the line-shape of  $\nu(\omega - \omega_0)$  becomes symmetric with respect to  $\omega = \omega_0$  and its maximum approaches the value  $\nu_b$  (see upper curve),

$$\nu_b = \left( \frac{\nu_l}{\pi} \int_0^\infty d\omega_t Q(\omega_t) \frac{W(\sqrt{2M\omega_t/\hbar})}{W(0)} \right)^{1/2}. \quad (11)$$

It should be emphasized that the described behavior of  $\nu(\omega)$ , as  $\omega_W$  is decreased, is a characteristic prediction of the self-consistent approach, which takes into account the action  $\nu$  on itself. Indeed, within the routine Born approximation Eq. (7) would reduce to a similar expression, but with  $\nu = 0$  in the resonant function (8). In this case, the line-shape of  $\nu(\omega)$  would be determined by the relation between  $\min\{\omega_W, \omega_Q\}$  and  $\nu_0$  alone. So, for small values of  $\nu_0$  ( $\nu_0 \ll \nu$ ) the transition from the sharp resonance to the broad one in the Born approximation can occur only at extremely large values of the correlation radius  $R_c$ . On the contrary, the self-consistent approach predicts a rapid transition to the broad resonance, which can be observed even for  $R_c$  of the order of the exciton Bohr radius  $a_0$ . This is due to the fact that, with increasing  $R_c$ , not only  $\omega_W$  is decreased, but also the scattering frequency  $\nu$  is substantially enhanced because of its self-action.

Now, we shall study how the exciton–surface scattering alters the absorptivity spectrum of a surface-corrugated deep quantum well. Neglecting the relatively small energy associated with light scattered in directions different from those of transmission and reflection [10], the quantum-well average absorptivity  $A$  is defined as  $A = 1 - R - T$ . Within the same approximation, the average reflectivity  $R$  and transmittivity  $T$  can be calculated from boundary conditions for

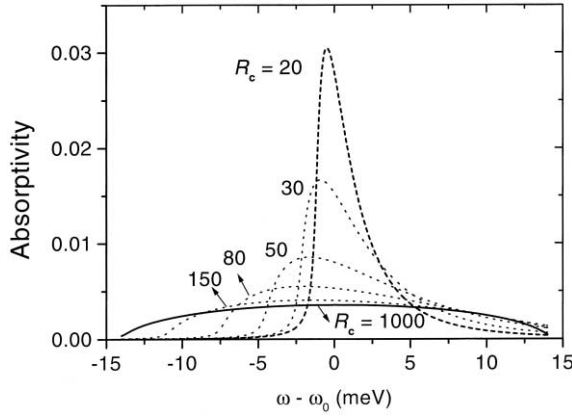


Fig. 2. Exciton absorptivity for the quantum wells considered in Fig. 1 vs. the resonance detuning  $\omega - \omega_0$ .

the exciton–polariton fields, obeying Maxwell’s equations with the ensemble-averaged excitonic polarization

$$\begin{aligned} \vec{P}(\vec{R}, Z, \omega) = & -2 \int d\vec{\rho} dz \mathcal{M}(\vec{\rho}, z) \int d\vec{R}' d\vec{\rho}' dz'_e dz'_h \mathcal{G}(\vec{R} \\ & - \vec{R}'; \vec{\rho}, \vec{\rho}'; z_e, z'_e; z_h, z'_h) \mathcal{M}(\vec{\rho}', z') \vec{E}(\vec{R}', Z', \omega). \end{aligned} \quad (12)$$

Here  $z = z_e - z_h$  and  $Z = (m_e z_e + m_h z_h)/M$ . The interband-transition dipole density  $\mathcal{M}(\vec{\rho}, z)$  is well described by the shell model [28–30],  $\mathcal{M}(\vec{\rho}, z) = \mathcal{M}_0 \delta(z) \delta(\rho - \rho_0) / 2\pi\rho_0$  with  $\rho_0 \rightarrow 0$ . Assuming that the light of frequency  $\omega$  with linear polarization in  $y$ -direction is normally incident upon the quantum-well plane and using Eqs. (6) and (12), the *nonlocal* relation between the excitonic polarization  $\vec{P}$  and the electric field  $\vec{E}$  acquires the form

$$\begin{aligned} P_y(Z, \omega) = & -\frac{64\mathcal{M}_0^2}{\pi d^2 a_0} \frac{\sin^2(\pi Z/d)}{\hbar(\omega - \omega_0 + i\nu_0 + i\nu)} \\ & \times \int_0^d dZ' E_y(Z', \omega) \sin^2(\pi Z'/d). \end{aligned} \quad (13)$$

As follows from this analytical result, the above introduced exciton–surface scattering frequency  $\nu \equiv \nu(k_x = 0)$  is precisely the quantity that determines the surface-induced broadening of the exciton ground resonance. This conclusion is confirmed by our theoretical curves in Fig. 2 for the absorptivity  $A(\omega)$  of quantum wells as those considered in Fig. 1. In complete accordance with the behavior of  $\nu(\omega)$ , the transition from the sharp to the broad resonance is well observed and the asymmetric line-shape of  $A(\omega)$  becomes symmetric. Note that beginning from  $R_c \sim 100 \text{ \AA}$ , the resonance in the absorptivity looks very broad. As was mentioned above, such a rapid transition to the broad resonance is a direct consequence of the self-consistent approach. We recall that within the usual Born approximation, this transition can be observed only at extremely large

values of the correlation length  $R_c$  (see, for example, Fig. 6 in Refs. [6]). Another interesting feature of  $A(\omega)$  is the behavior of its maximum. In the regime of the sharp resonance, it undergoes a red shift as  $R_c$  is increased. Then, in the opposite regime (broad resonance), the maximum of  $A(\omega)$  is shifted back, approaching the resonance point  $\omega = \omega_0$ . We stress here that such a nonmonotonic behavior of the absorptivity maximum is caused by the self-action of the exciton–surface scattering frequency  $\nu(\omega)$ .

#### 4. Summary

We have applied the self-consistent Green’s function method to derive and analyze the exciton–surface scattering frequency  $\nu$  in quantum wells. We have proved that the relation between the total scattering frequency  $\nu_0 + \nu$  and the scaling frequency  $\min\{\omega_W, \omega_Q\}$  controls the line-shape of  $\nu(\omega)$  and, consequently, of the optical absorptivity  $A(\omega)$ . These line-shapes are asymmetric in the case of the sharp resonance ( $\nu_0 + \nu \ll \min\{\omega_W, \omega_Q\}$ ) and become symmetric at the broad resonance ( $\min\{\omega_W, \omega_Q\} \ll \nu_0 + \nu$ ). As  $R_c$  is increased, the behavior of the positions of the maxima for  $\nu(\omega)$  and  $A(\omega)$  turns out to be nonmonotonic. The above conclusions straightforwardly follow from Eq. (7) and, therefore, are valid for arbitrary height of barrier. We found the explicit dependence of  $\nu$  on other parameters of the exciton quantum well and interface roughness. For the case of deep wells ( $b_{e,h} \approx 1$ ) and sharp resonance  $\nu \sim \nu_l \propto d^{-6} \zeta^2$  (Eq. (10)), whereas for the broad resonance  $\nu \sim \nu_b \propto d^{-3} \zeta$  (Eq. (11)). Surprisingly, the latter dependence of  $\nu$  on  $\zeta$  is linear. Evidently, our results do not change qualitatively in the general case of an anisotropic exciton, different effective masses for the electron and hole within the quantum well and barrier, and the geometry of oblique incidence of light. The method developed here can be easily extended to quantum wells with arbitrary statistical properties of their interfaces.

#### Acknowledgements

This work was partially supported by CONACYT under Grant No. 26184-E. N.M.M. acknowledges support from CONACYT.

#### References

- [1] C. Weisbuch, R. Dingle, A.C. Gossard, W. Wiegmann, *Solid State Commun.* 38 (1981) 709.
- [2] M. Tanaka, H. Sakaki, *J. Cryst. Growth* 81 (1987) 153.
- [3] R.F. Kopf, E.F. Schubert, T.D. Harris, R.S. Becker, *Appl. Phys. Lett.* 58 (1991) 631.
- [4] R. Zimmermann, *Phys. Stat. Sol. B* 173 (1992) 129.
- [5] S. Glutsch, F. Bechstedt, *Phys. Rev. B* 50 (1994) 7733.

- [6] S. Glutsch, D.S. Chemla, F. Bechstedt, *Phys. Rev. B* 54 (1996) 11592.
- [7] Al. L. Efros, C. Wetzel, *Phys. Rev. B* 52 (1995) 8384.
- [8] T. Stroucken, A. Knorr, C. Anthony, A. Schulze, P. Thomas, S.W. Koch, M. Koch, S.T. Cundiff, J. Feldmann, E.O. Göbel, *Phys. Rev. Lett.* 74 (1995) 2391.
- [9] U. Jahn, S.H. Kwok, M. Ramsteiner, R. Hey, H.T. Grahn, E. Runge, *Phys. Rev. B* 54 (1996) 2733.
- [10] L.C. Andreani, G. Panzarini, A.V. Kovokin, M.R. Vladimirova, *Phys. Rev. B* 57 (1998) 4670.
- [11] V.A. Kosobukin, *Solid State Commun.* 108 (1998) 83.
- [12] N.T. Pelekanos, N. Boudet, J. Eymery, H. Mariette, J. Cryst. Growth 184/185 (1998) 886.
- [13] E. Runge, R. Zimmermann, *Phys. Stat. Sol. B* 206 (1998) 167.
- [14] J.B.B. de Oliveira, E.A. Meneses, E.C.F. da Silva, *Phys. Rev. B* 60 (1999) 1519.
- [15] S. Das Sarma, D.W. Wang, *Phys. Rev. Lett.* 84 (2000) 2010.
- [16] G. Malpuech, A. Kavokin, W. Langbein, J.M. Hvam, *Phys. Rev. Lett.* 85 (2000) 650.
- [17] P. Halevi (Ed.), *Spatial Dispersion in Solids and Plasmas Electromagnetic Waves — Recent Developments in Research*, vol. 1, Elsevier, Amsterdam, 1992 (Chapter 6).
- [18] G.S. Agarwal, C.V. Kunasz, *Phys. Rev. B* 26 (1982) 5832.
- [19] E.L. Alburquerque, N.S. Almeida, M.C. Oliveros, *Phys. Stat. Sol. B* 129 (1985) 177.
- [20] G.H. Coccoletzi, S. Wang, *Phys. Rev. B* 48 (1993) 17413.
- [21] V.A. Kosobukin, A.V. Sel'kin, *Solid State Commun.* 66 (1988) 313.
- [22] V.A. Kosobukin, M.I. Sazhin, A.V. Sel'kin, *Solid State Commun.* 94 (1995) 947.
- [23] J. Madrigal-Melchor, F. Pérez-Rodríguez, A. Silva-Castillo, H. Azucena-Coyotécatl, *Fiz. Tverd. Tela (St. Petersburg)* 40 (1998) 865 [*Phys. Solid State* 40 (1998) 796].
- [24] J. Madrigal-Melchor, H. Azucena-Coyotécatl, A. Silva-Castillo, F. Pérez-Rodríguez, *Phys. Rev. B* 61 (2000) 15993.
- [25] L.D. Landau, E.M. Lifshitz, *Quantum Mechanics*, Pergamon Press, Oxford, 1974.
- [26] F.G. Bass, I.M. Fuks, *Wave Scattering from Statistically Rough Surfaces*, Pergamon Press, New York, 1979.
- [27] A.R. McGurn, A.A. Maradudin, *Phys. Rev. B* 30 (1984) 3136.
- [28] A. Stahl, I. Balslev, *Phys. Stat. Sol. B* 113 (1982) 583.
- [29] G. Czajkowski, A. Tredicucci, *Nuovo Cimento D* 14 (1992) 1203.
- [30] D. Merbach, E. Schöll, W. Ebeling, P. Michler, J. Gutowski, *Phys. Rev. B* 58 (1998) 10709.
- [31] L.C. Andreani, F. Bassani, *Phys. Rev. B* 41 (1990) 7536.
- [32] F. Tassone, F. Bassani, L.C. Andreani, *Phys. Rev. B* 45 (1992) 6023.