Selective transport and mobility edges in quasi-one-dimensional systems with a stratified correlated disorder

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We present analytical results on transport properties of many-mode waveguides with randomly stratified disorder having long-range correlations. To describe such systems, the theory of one-dimensional transport recently developed for a correlated disorder is generalized. The propagation of waves through such waveguides may reveal a quite unexpected phenomena of a complete transparency for a subset of propagating modes. We found that with a proper choice of long-range correlations one can arrange a perfect transparency of waveguides inside a given frequency window of incoming waves. Thus, mobility edges are shown to be possible in quasi-one-dimensional geometry with correlated disorder. The results may be important for experimental realizations of a selective transport in application to both waveguides and electron/optic nanodevices. © *2004 American Institute of Physics*. [DOI: 10.1063/1.1765735]

During the last few years there are a burst of interest to the problem of localization–delocalization transition in systems with correlated disorder (see, e.g., Refs. 1 and 2, and references therein). This fact is due to the possibility to observe an anomalous transport in one-dimensional (1D) devices with random potentials. In particular, it was shown[2] that specific long-range correlations give rise to a complete transparency of electron waves for given energy intervals. Experimental realization of such potentials for single-mode waveguides with delta-like scatters[3] has confirmed theoretical predictions.

The subject of wave propagation through disordered stratified media is important both from the theoretical viewpoint and for experimental applications such as optic fibers, remote sensing, radio wave propagation, shallow water waves, random photonic lattices, etc. (see, e.g., Ref. 4). It has also a direct relation to the problem of electronic transport in mesoscopic conducting channels. So far, main results in this field have been obtained for random Gaussian potentials. On the other hand, the understanding of generic properties of transport in the systems with *correlated disorder* is important for the modern theory of electron/wave propagation. It should also be stressed that existing experimental technics allow for the construction of systems with sophisticated scattering potentials resulting in anomalous transport properties[5].

In this letter we study transport properties of quasi-1D waveguides with a *stratified* disorder that has long-range correlations. Our main interest is in exploring the possibility of constructing such random potentials that result in frequency windows of a perfect propagation of waves.

In what follows we consider a plane waveguide (or conducting wire) of width d having a stratified region of the length L. The stratification is described by a random potential $V(x)$ with zero average $\langle V(x)\rangle=0$, and correlator $\langle V(x)V(x')\rangle=\mathcal{W}(|x-x'|)$. The x axis stretches along the waveguide, the angular brackets stand for the statistical average over realizations of $V(x)$[6].

Since the potential $V(x)$ does not depend on the transverse to the waveguide direction, the *total transmittance* $T(L)$ of a quasi-1D stratified structure is expressed as a sum of *partial transmittances* T_n,

$$T(L)=\sum_{n=1}^{N_d} T_n(L). \tag{1}$$

Here $N_d=[kd/\pi]$ is the total number of propagating modes (*conducting channels*) with $[\dots]$ as the integer part, and k is the wave number. Every T_n describes the transparency of the corresponding nth propagating mode with a lengthwise wave number $k_n=\sqrt{k^2-(\pi n/d)^2}$. In such a way our problem for the quasi-1D disordered system has reduced to the 1D wave scattering in every nth channel. As a result, the average partial transmittance $\langle T_n\rangle$ is described by the universal function that depends on one parameter $\Lambda_n\equiv L/L_{\rm loc}(k_n)$ only, where $L_{\rm loc}(k_n)$ is the *localization length* associated with a given channel.

According to the theory of 1D random media,[7,8] for a weak potential (and not close to the energy band $k=0$) the inverse localization length is given by

$$L_{\rm loc}^{-1}(k_n)=W(2k_n)/16k_n^2, \tag{2}$$

where $W(k_x)$ is the Fourier transform of the binary correlator $\mathcal{W}(|x-x'|)$ for the scattering potential. For large localization length, $\Lambda_n=L/L_{\rm loc}(k_n)\ll 1$, the transmittance $\langle T_n\rangle$ exhibits the ballistic behavior and the corresponding nth normal mode is practically transparent, $\langle T_n\rangle\approx 1-4\Lambda_n$. On the contrary, the transmittance is exponentially small, $\langle T_n\rangle\sim\Lambda_n^{-3/2}\exp(-\Lambda_n)$, when the localization length is much less than the length of the waveguide, $\Lambda_n\gg 1$. This implies strong electron/wave localization in a given nth channel.

The main feature of $L_{\rm loc}(k_n)$ is its dependence on the mode index n. One can see that the larger n is, the smaller is

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$L_{\rm loc}(k_n)$ and, consequently, the stronger is the coherent scattering within this mode. This dependence is quite strong due to the squared wave number k_n in the denominator of Eq. (2). Evidently, with an increase of n the value of k_n decreases. An additional dependence appears because of the power spectrum $W(2k_n)$. Since the correlator $\mathcal{W}(|x-x'|)$ is a decreasing function of $|x-x'|$, the numerator $W(2k_n)$ increases with n (it is a constant for the delta-correlated potential only). Therefore, *both* the numerator and denominator contribute in the same way for the dependence of $L_{\rm loc}(k_n)$ on n. As a result, we arrive at the *hierarchy of mode localization lengths*,

$$L_{\rm loc}(k_{N_d}) < L_{\rm loc}(k_{N_d-1}) < \cdots < L_{\rm loc}(k_2) < L_{\rm loc}(k_1).$$

Thus, a remarkable phenomenon arises. On the one hand, the partial transport for any of N_d channels is characterized solely by Λ_n. On the other hand, due to the revealed hierarchy of $L_{\rm loc}(k_n)$, the *total* transmittance (1) depends on the whole set of scaling parameters Λ_n. This fact is in contrast with quasi-1D bulk-disordered models, for which all transport properties are described by one parameter only. Note that our model is similar to quasi-1D waveguides with rough surfaces, for which an hierarchy of scattering lengths was also found.[9]

The combined effect of the hierarchy of $L_{\rm loc}(k_n)$ and the one-parameter scaling for every $\langle T_n \rangle$ results in three different transport regimes. (i) If the largest localization length is much less than the waveguide length, $L_{\rm loc}(k_1) \ll L$, all propagating modes are localized and the waveguide is nontransparent. (ii) On the contrary, when the smallest localization length $L_{\rm loc}(k_{N_d})$ is much larger than L, all propagating modes are open ($\langle T_n \rangle \approx 1$) and the total ballistic transmittance is equal to the total number of propagating modes, $\langle T(L) \rangle = N_d$. (iii) The intermediate situation arises when $L_{\rm loc}(k_1)$ is larger, while $L_{\rm loc}(k_{N_d})$ is smaller than the waveguide length, $L_{\rm loc}(k_{N_d}) \ll L \ll L_{\rm loc}(k_1)$. In this case a very interesting phenomenon of the *coexistence* of *ballistic* and *localized* transport occurs. Namely, while *lowest* modes are in the ballistic regime, *highest* modes are strongly localized.

As a demonstration, let us consider the waveguide with a large number of conducting channels, $N_d = [kd/\pi] \approx kd/\pi \gg 1$, and with potential having the widely used Gaussian correlator,

$$\mathcal{W}(|x|) = W_0 k_0 \pi^{-1/2} \exp(-k_0^2 x^2), \qquad W(k_x) = W_0 \exp(-k_x^2/4k_0^2). \tag{3}$$

It is convenient to introduce two parameters

$$\alpha \equiv \frac{L}{L_{\rm loc}^w(k_1)} = \frac{W_0 L}{16k^2}, \quad \delta \equiv \frac{L_{\rm loc}^w(k_{N_d})}{L_{\rm loc}^w(k_1)} \ll 1, \tag{4}$$

where $L_{\rm loc}^w(k_1)$ and $L_{\rm loc}^w(k_{N_d})$ refer, respectively, to the largest and smallest localization lengths in the limit case of the white-noise potential ($k_0 \to \infty$).

One can find that for Gaussian correlations (3) all propagating modes are localized when $\alpha \gg \exp(k^2/k_0^2)$. The intermediate situation occurs when

$$\exp(k^2/k_0^2) \gg \alpha \gg \delta \exp(\delta k^2/k_0^2). \tag{5}$$

Therefore, the longer range k_0^{-1} of the correlated disorder, the simpler the conditions (5) of the *coexistence* of ballistic and localized transport. Finally, the waveguide is transparent in the case when $\alpha \ll \delta \exp(\delta k^2/k_0^2)$. One can show that for white-noise potentials the condition $\alpha \gg 1$ localizes all propagating modes. However, with a proper choice of the stratified correlated disorder the ballistic transport of all conducting channels can be realized even for $\alpha \gg 1$.

The fundamentally different situation arises when the stratified medium has specific long-range correlations. To show this, we note that the localization length $L_{\rm loc}(k_n)$ of any nth conducting channel is entirely determined by $W(k_x)$; see Eq. (2). Therefore, if $W(2k_n)$ vanishes for some wave numbers k_n, then $L_{\rm loc}(k_n)$ diverges and corresponding propagating modes are fully transparent. Such a situation can be realized by using the method of constructing random potentials with a given $W(k_x)$.[2,10] Specifically, one should find the function $\beta(x)$ whose Fourier transform is $W^{1/2}(k_x)$. Then, the random potential $V(x)$ can be constructed as a convolution of white noise $Z(x)$ with the function $\beta(x)$,

$$V(x) = \int_{-\infty}^{\infty} dx'\ Z(x-x')\ \beta(x'). \tag{6}$$

We emphasize that the transition between localized and ballistic wave/electron transport can be arranged in an *abrupt way* at any given value of wave number k_n. This can be done if $W(k_x)$ has a discontinuity at the desired point. Correspondingly, random potential $V(x)$ should have *long-range correlations*.

To show how to arrange a sharp transition, let us take $V(x)$ with the following power spectrum:

$$W(k_x) = W_0\ \Theta(|k_x| - 2k_0), \tag{7}$$

where $\Theta(x)$ is the unit-step function and the characteristic wave number $k_0 > 0$ is a correlation parameter to be specified. According to Eq. (6), the random potentials having such correlator can be constructed as

$$V(x) = W_0^{1/2} \left[Z(x) - \int_{-\infty}^{\infty} dx'\ Z(x-x') \frac{\sin(2k_0 x')}{\pi x'} \right].$$

Now one can see that all low modes with wave numbers $k_n > k_0$ have finite localization lengths while for high modes with $k_n < k_0$ the localization lengths diverge,

$$L_{\rm loc}^{-1}(k_n > k_0) = W_0/16k_n^2, \quad L_{\rm loc}^{-1}(k_n < k_0) = 0. \tag{8}$$

Remarkably, in contrast to the case of Gaussian correlations (see above), here all propagating modes with

$$n > N_{\rm loc} = [(kd/\pi)(1 - k_0^2/k^2)^{1/2}]\Theta(k - k_0) \tag{9}$$

are ballistic. Since their mode transmittance $T_n = 1$, such modes form a *coset of completely transparent channels*. As for other propagating modes with *low* indices $n \leqslant N_{\rm loc}$, they remain to be *localized* for large enough waveguide lengths L when $W_0 L/16k_1^2 \gg 1$.

The expression (9) determines the total number $N_{\rm loc}$ of *localized modes* and total number $N_{\rm tr} = N_d - N_{\rm loc}$ of *completely transparent modes*. Since localized modes do not contribute to the total transmittance (1), the latter is equal to the number $N_{\rm tr}$ and do not depend on L,

$$\langle T \rangle = [kd/\pi] - [(kd/\pi)(1 - k_0^2/k^2)^{1/2}]\Theta(k - k_0). \tag{10}$$

For $k_0 \ll k$ the number of localized modes $N_{\rm loc} \approx [(kd/\pi)(1 - k_0^2/2k^2)]$ is of the order of N_d. Consequently, the number of transparent modes $N_{\rm tr}$ is small, or there are no such modes at all. Otherwise, if $k_0 \to k$, the integer $N_{\rm loc} \approx [\sqrt{2}(kd/\pi)(1-k_0/k)^{1/2}]$ turns out to be much less than the total number of waveguide modes N_d, and $N_{\rm tr}$ is large. When $k_0 > k_1$, the number $N_{\rm loc}$ vanishes and *all modes* become fully transparent. In this case the correlated disorder results in a perfect transmission of waves. Thus, the value $k_0 = k_1$ is, in essence, the *total mobility edge* that separates the region of complete transparency from that where lower modes are localized.

It is worthwhile to emphasize a nonmonotonic stepwise dependence of the total transmittance (10) on the wave number k. Specifically, within the region $k_1 < k_0$ for fixed values of k_0 [when $W_0 L d^2/16\pi^2 > (k_0 d/\pi)^2 > 1$], all propagating modes are transparent and the transmittance exhibits stepwise increase with an increase of k. Each step up arises for an integer value of kd/π when new conducting channel emerges.

On the other hand, for $k_1 > k_0$ the transmittance reveals steps down due to successive localization of low modes. In contrast with the steps up, every step down occurs at the *local mobility edge* of the corresponding channel, when the second term in Eq. (10) takes an integer value. These values may not coincide with the integer values of kd/π, thus resulting in a new kind of stepwise dependence for the transmittance. Finally, for very large values $k^2 \gg W_0 L/16$ the transmittance starts to increase again due to a successive delocalization of the modes.

Our results can be used for fabrications of electron or optic nanodevices with a selective transport. For example, for the GaAs quasi-1D quantum-well structures with the effective electron mass $m_e = 6.7\times 10^{-2} m_0$, the Fermi-energy $E_F = 7$ meV and $d = 500$ nm, the number of channels is about 17. Therefore, if choose the potential with $\pi/k_0 \approx 80$ nm, with an increase of k one can observe in the conductance, after 6 steps up, about 16 local mobility edges characterized by the steps down.

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